Characterizing the Structure of Oscillating Systems

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Information & Decision Algorithms Laboratories

Outline

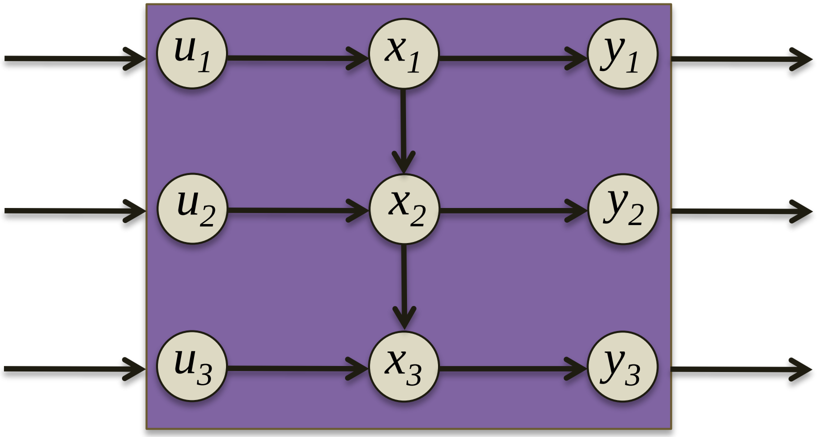
- Background: Studying the Structure of a Linear Time Invariant System
- Motivation: Many systems oscillate, so current tools won't work
- First Results: Lifting a Time-Varying System
- Next Idea: Decoupled sampling
- Conclusions
- Future Work

Structure of a Linear Time-Invariant System

Two Representations:

- State Space Realization
  - Physical interconnections
- Transfer Function
  - Input-output relations

Structure of a Linear Time-Invariant System



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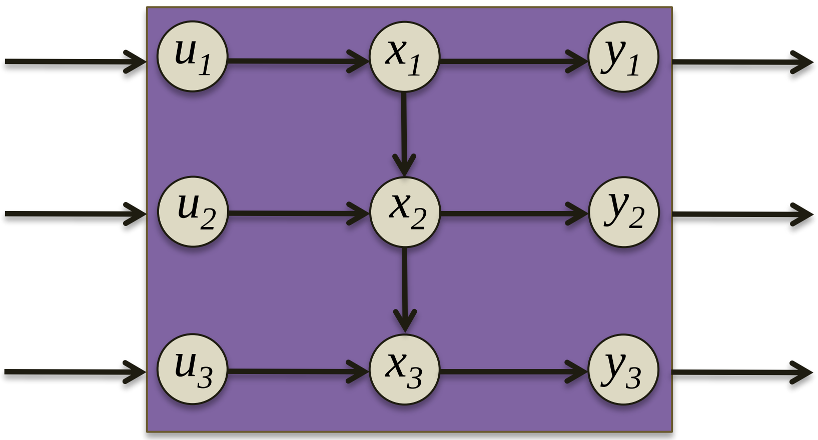

State Space Realization

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

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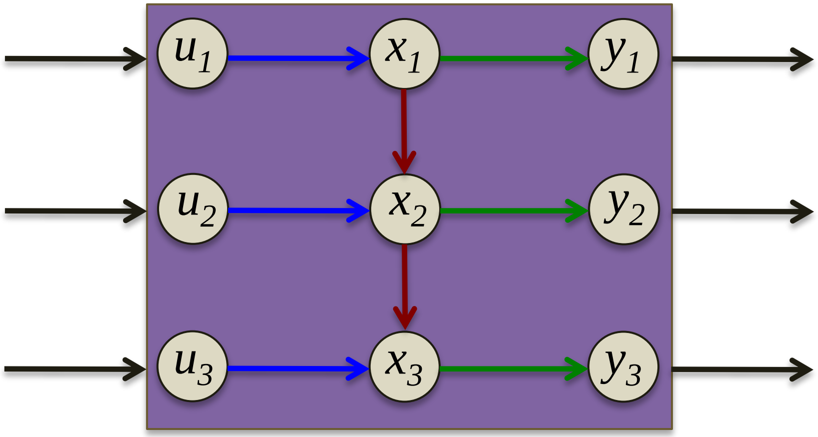

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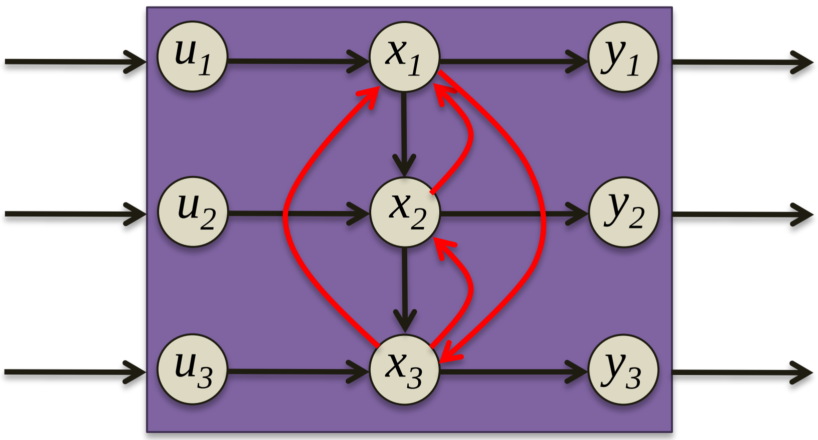

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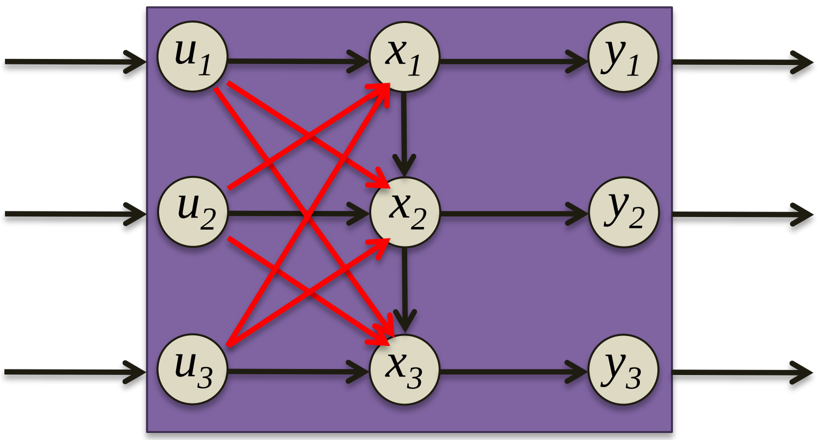

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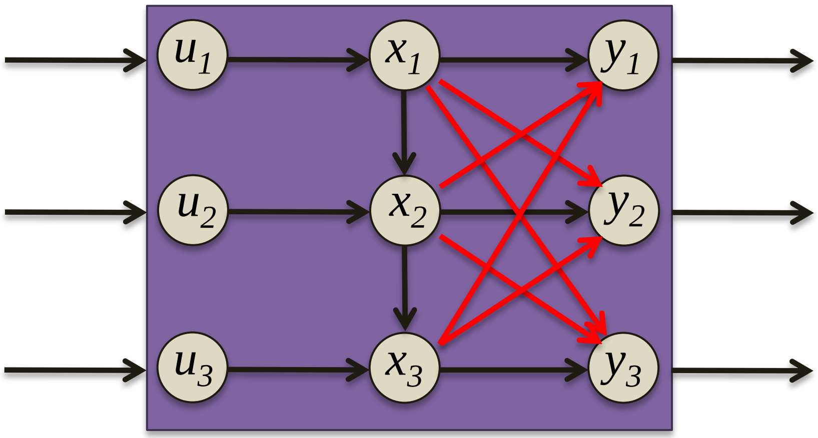

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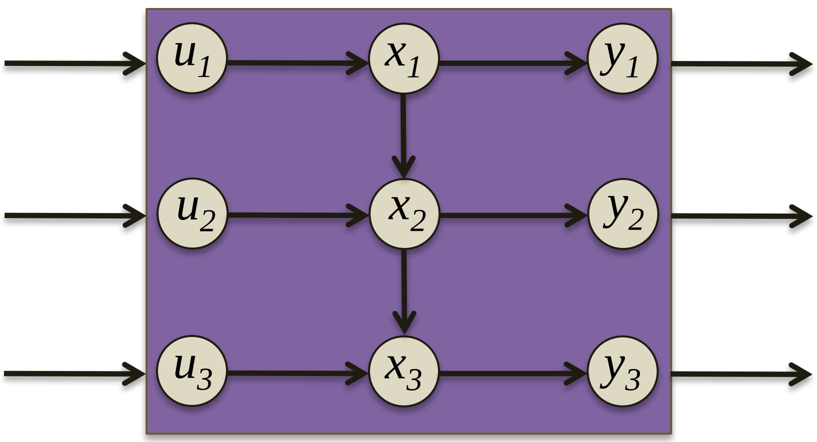

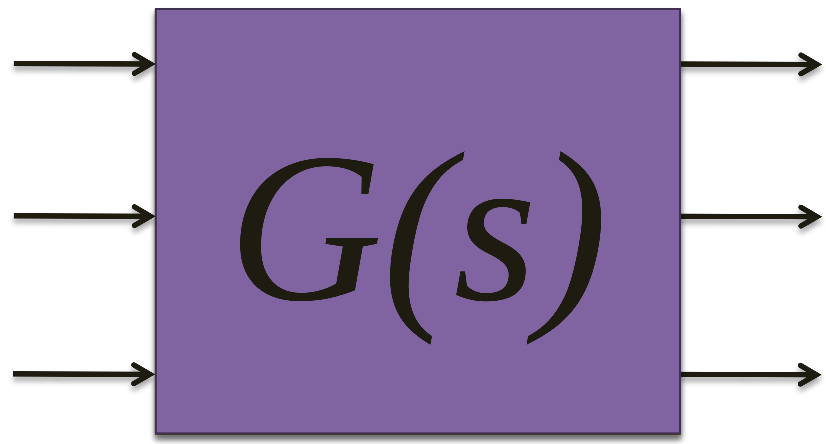

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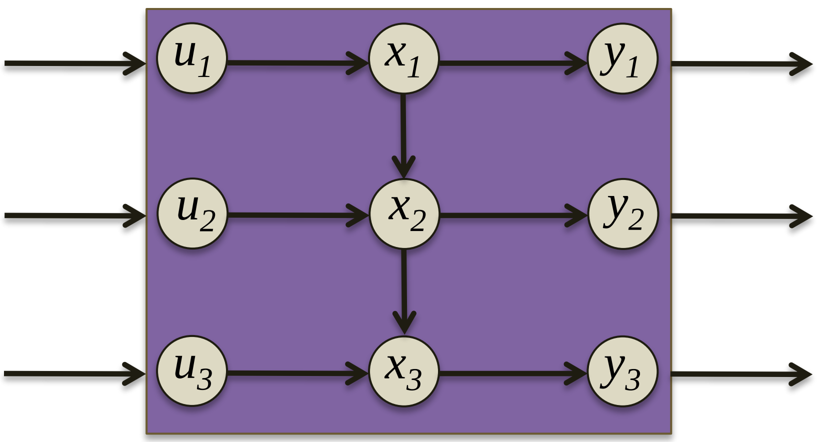

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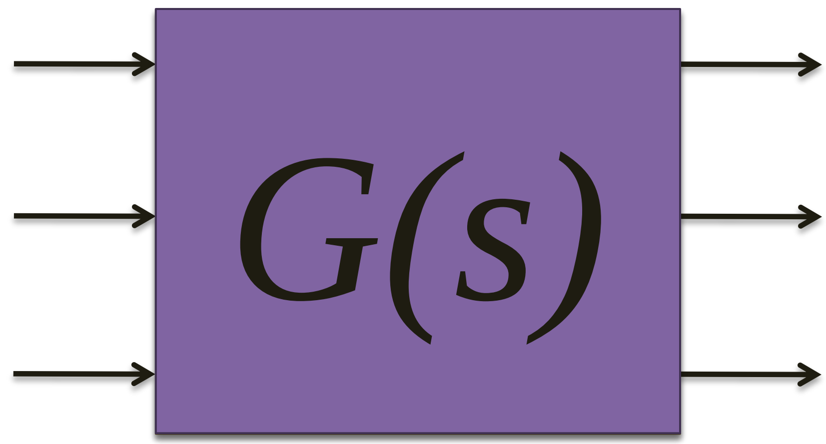

Transfer Function

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

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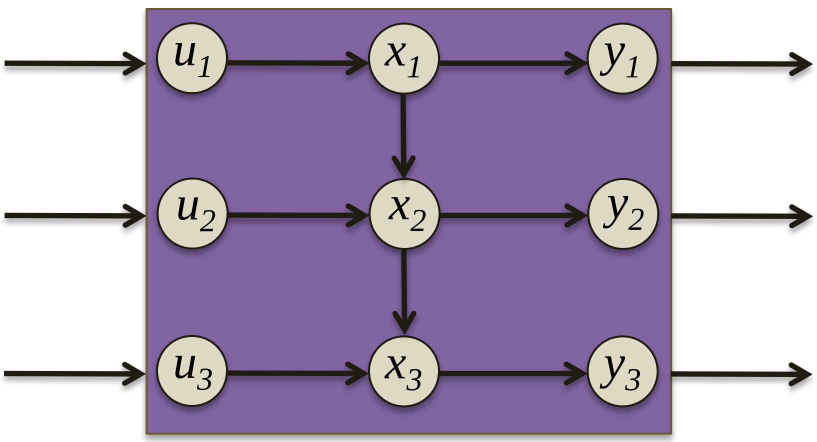

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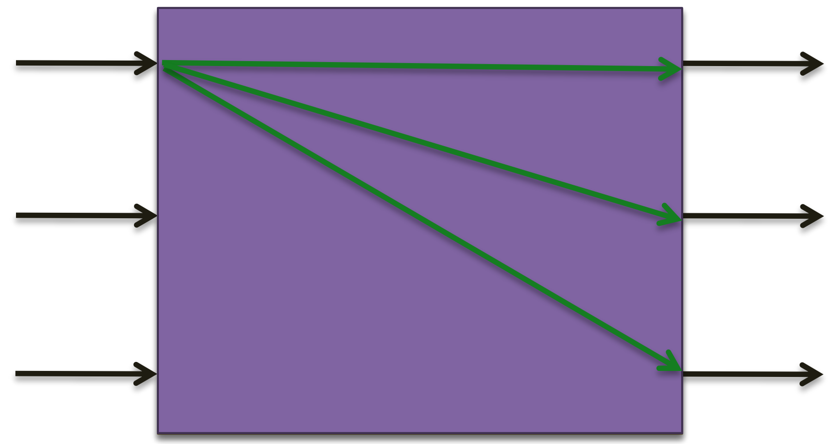

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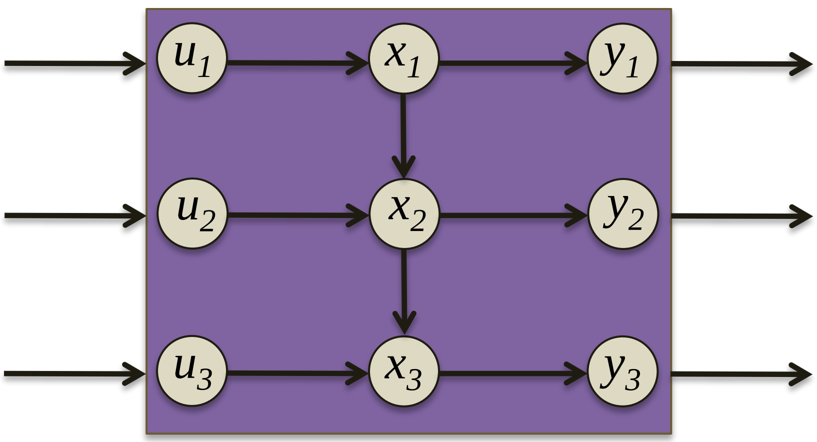

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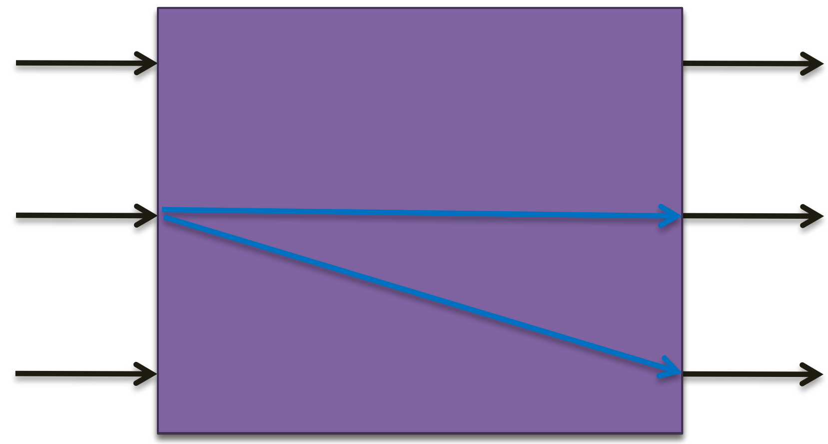

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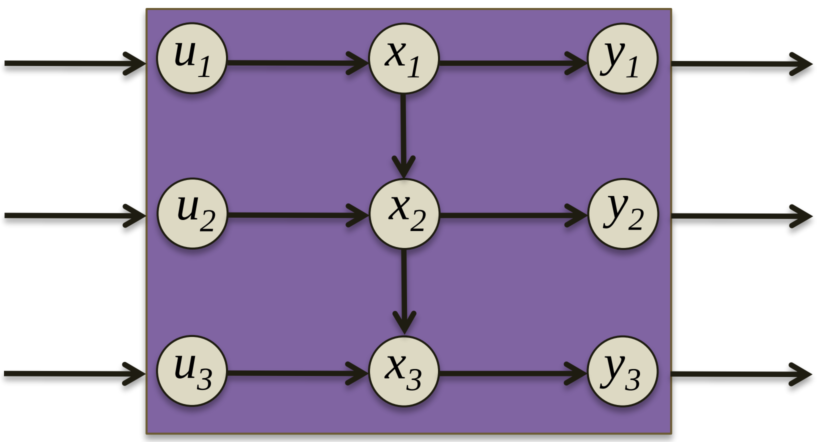

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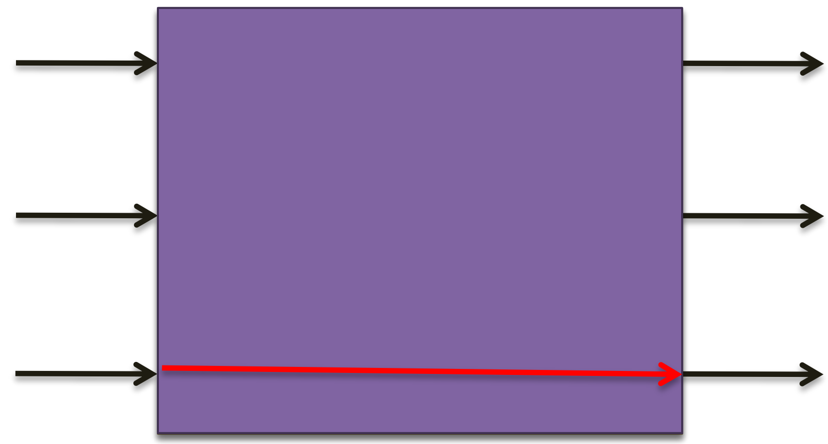

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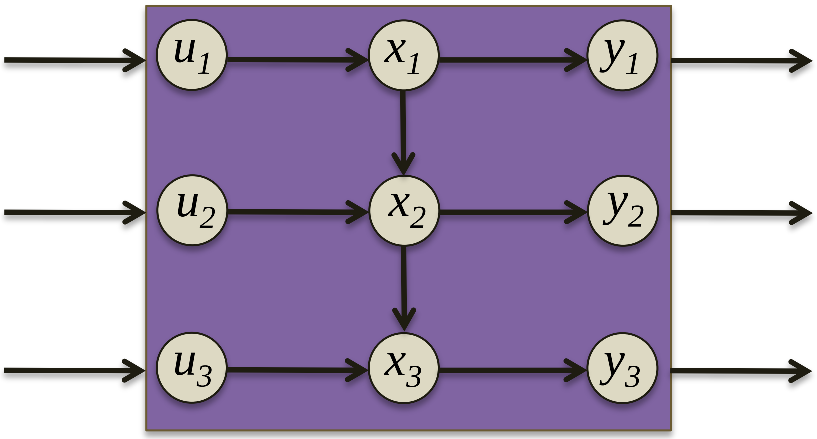

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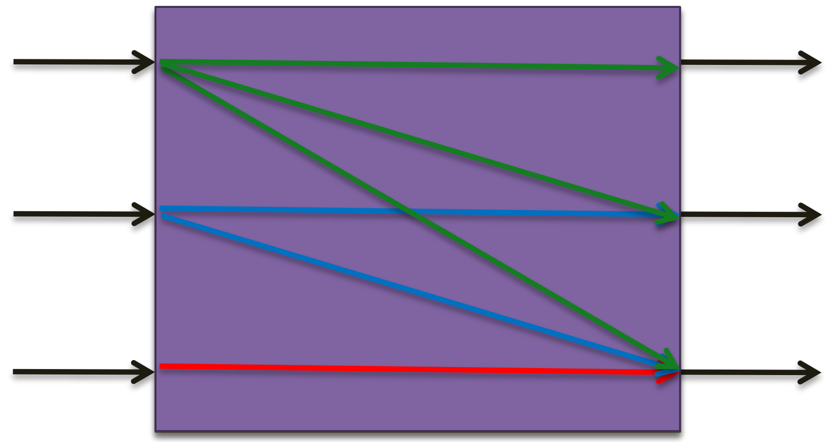

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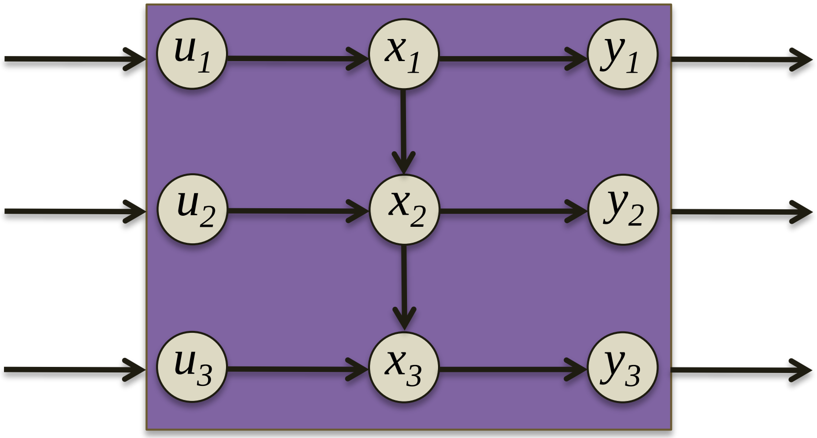

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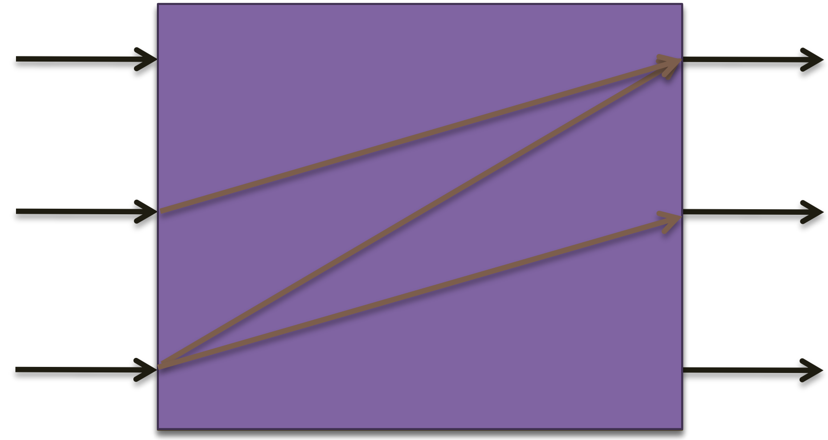

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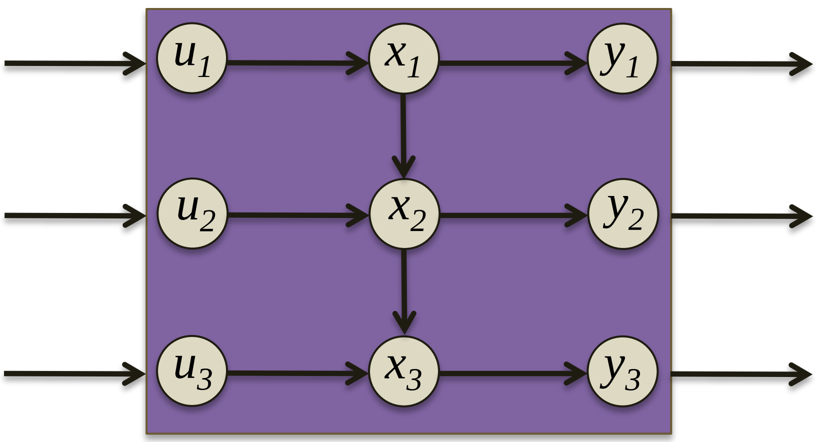

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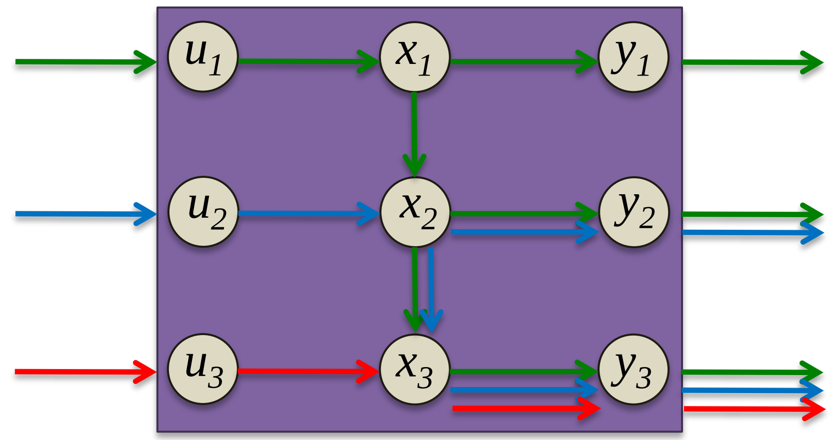

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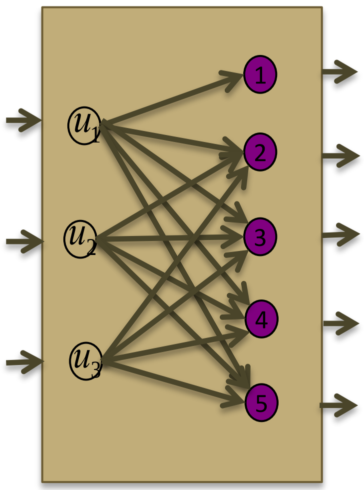
Dynamical Structure Functions

- Another way of representing a system
- Input-Output & Output-Output relations.
- More specific than the Transfer Function representation, but less specific than the State Space Realization.



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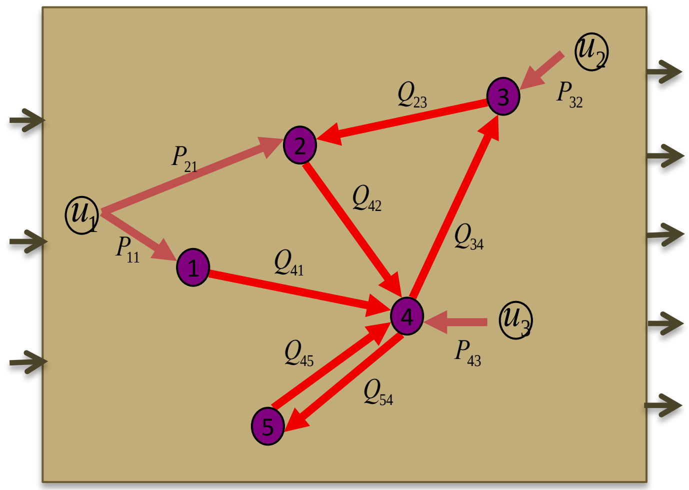

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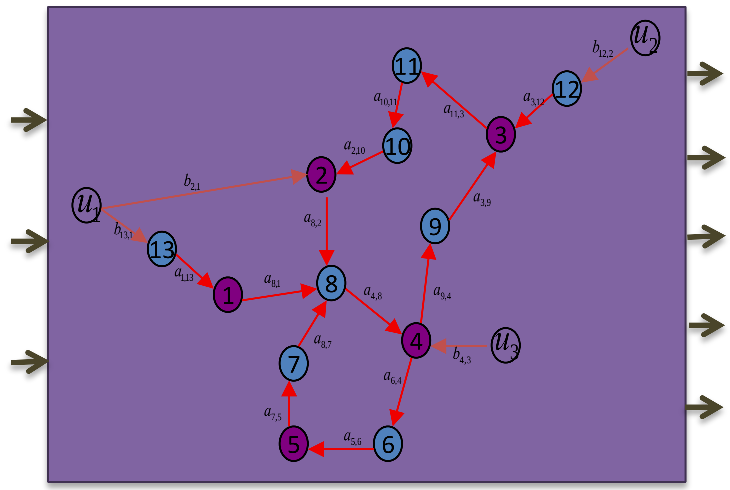

State Space Realization

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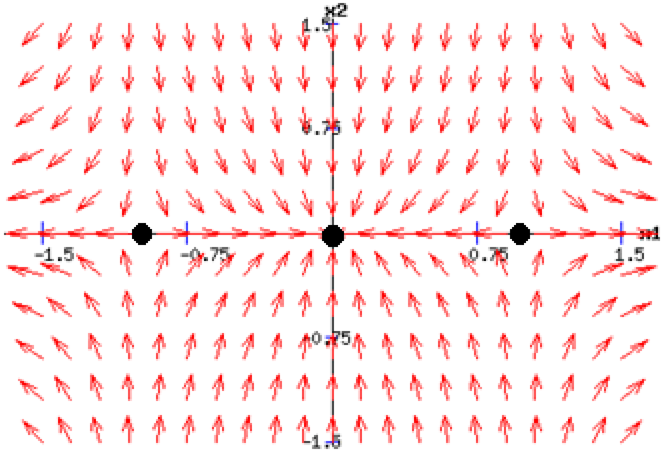
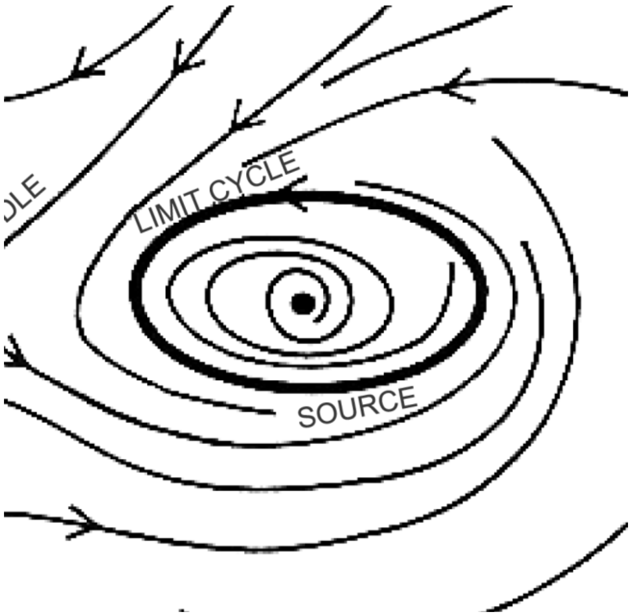
- These systems are actually nonlinear
  - Linearize near equilibrium point
  - Lyapunov: Structure of a linear system is the same structure of a nonlinear system near equilibrium point.
- Example: Homeostasis



Many Systems Oscillate

- Many biological systems oscillate
- Examples:
  - Breathing
  - Cardiovascular system
- These systems have no equilibrium point but rather a **limit cycle**
  - Linearize around limit cycle
  - Periodically Time Varying
- How to study structure?



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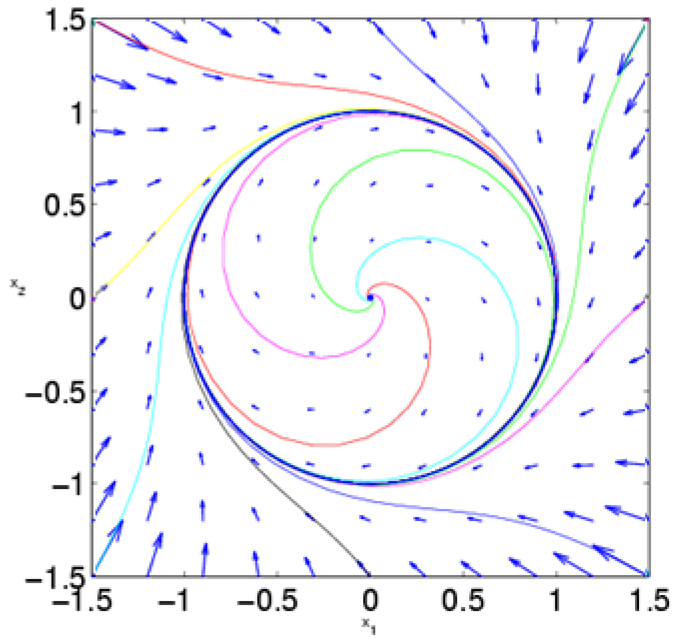
Example: Electronic Oscillator

- We want to study the structure of this system

$$\frac{dx_1}{dt} = x_2 + x_1\left(1 - x_1^2 - x_2^2\right)$$

$$\frac{dx_2}{dt} = -x_1 + x_2\left(1 - x_1^2 - x_2^2\right)$$



Example:
Electronic Oscillator

- We want to study the structure of this system
- Linearize around limit cycle

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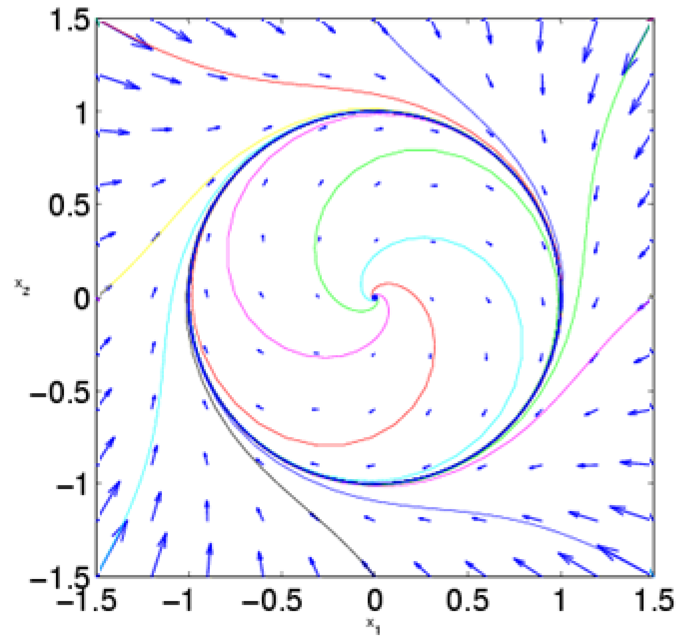
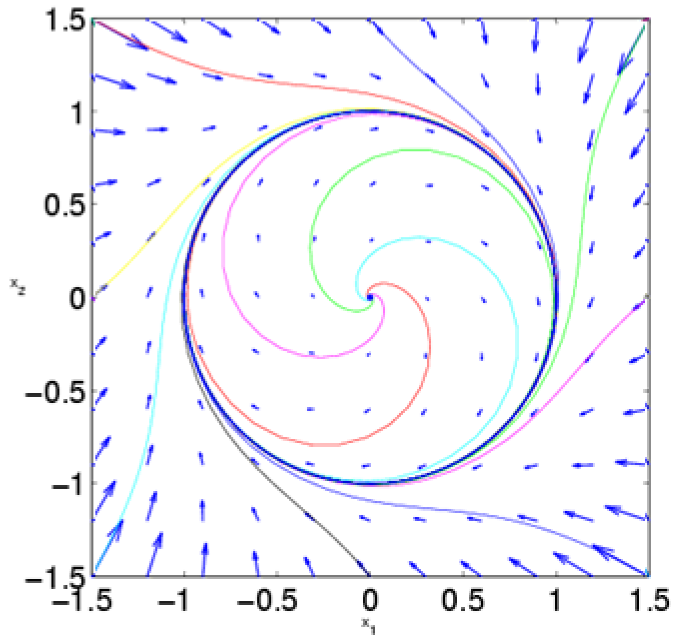


Example: Electronic Oscillator

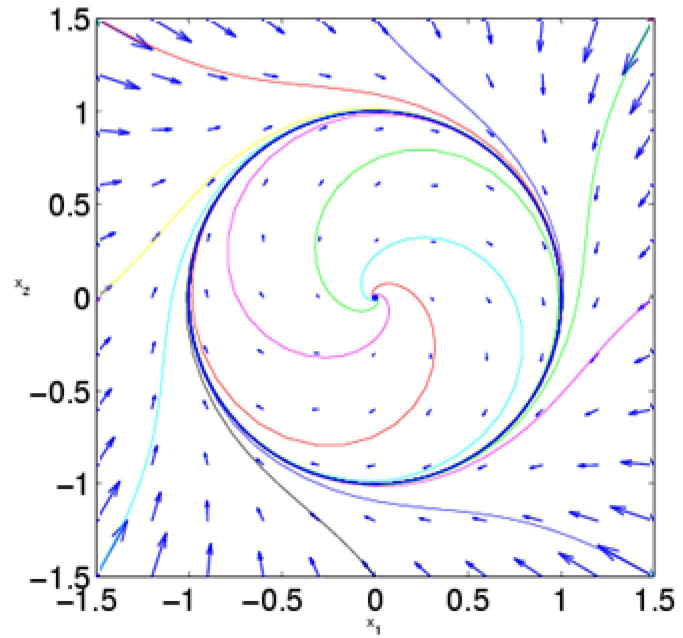
- We want to study the structure of this system
- Linearize around limit cycle

$$\begin{pmatrix} \dot{\delta_1}(t) \\ \dot{\delta_2}(t) \end{pmatrix} \approx \begin{pmatrix} -2sin^2(t) & 1-2sin(t)cos(t) \\ 1-2sin(t)cos(t) & -2cos^2(t) \end{pmatrix} \begin{pmatrix} \delta_1(t) \\ \delta_2(t) \end{pmatrix}$$



Example: Electronic Oscillator

- We want to study the structure of this system
- Linearize around limit cycle
- Need time invariant representation



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First Results: Lifting

- This method finds a time-invariant representation of a time varying system.
- Maintains some properties of the system

$$A_l = A(N-1)A(N-2)...A(0)$$

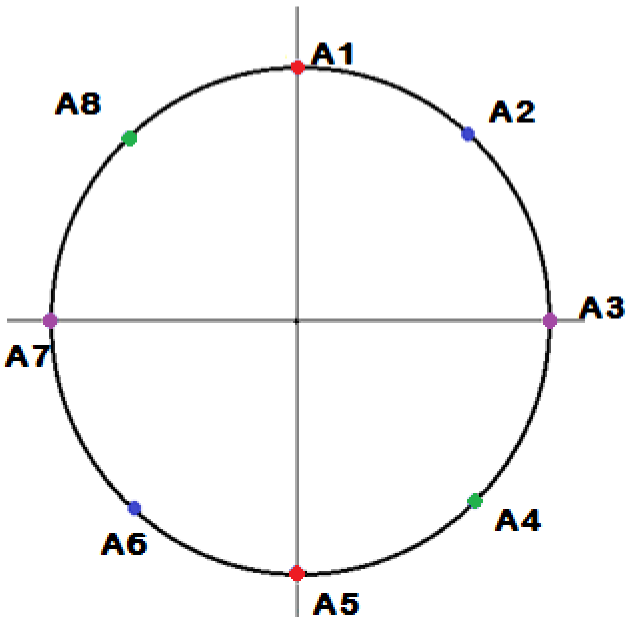
$$B_l = \begin{pmatrix} A(N-1)A(N-2)\dots A(1)B(0) & A(N-1)\dots A(2)B(1) & \dots & B(N-1) \end{pmatrix}$$

$$C_l = \begin{pmatrix} C(0) \\ C(1)A(0) \\ \vdots \\ C(N-1)A(N-2)\dots A(0) \end{pmatrix}$$

How to lift an N-periodic system.

Example

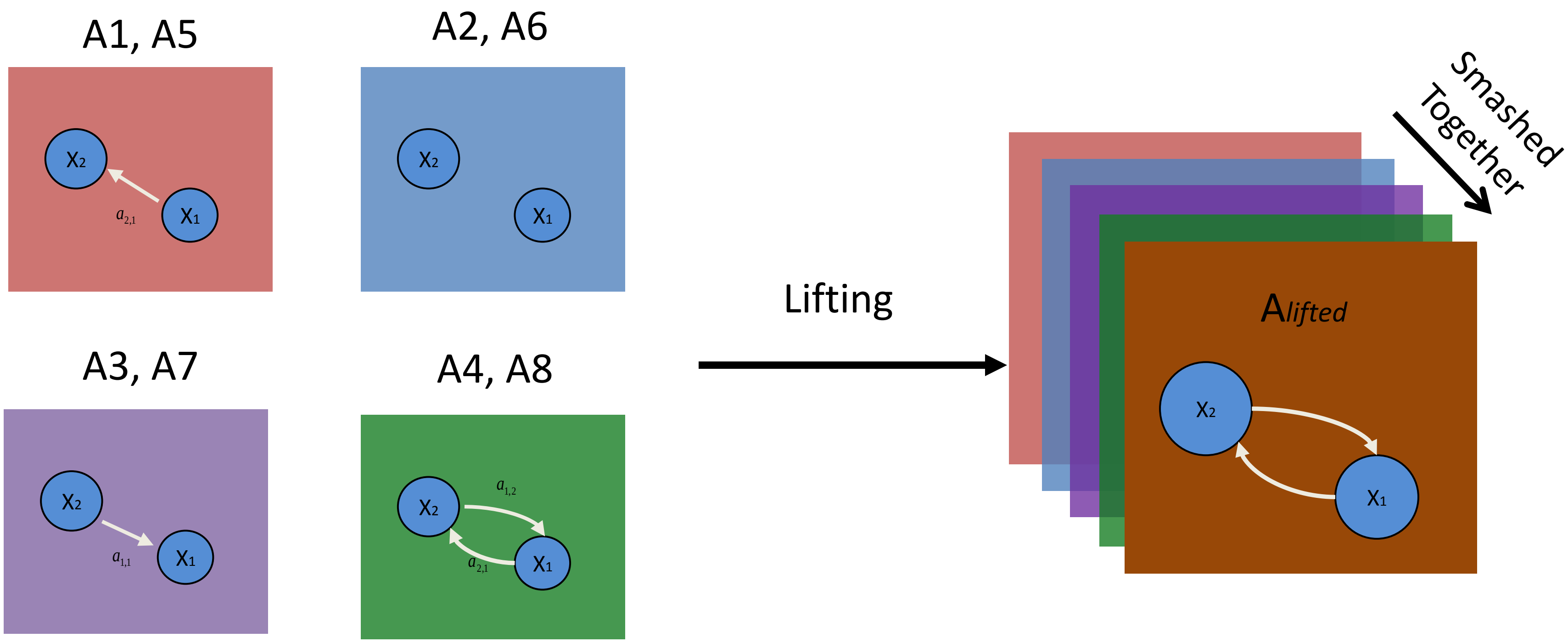
- Lifting the Oscillator



$$A1 = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$
$$A2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$A3 = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$$
$$A4 = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$
$$A5 = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$
$$A6 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$A7 = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$$
$$A8 = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

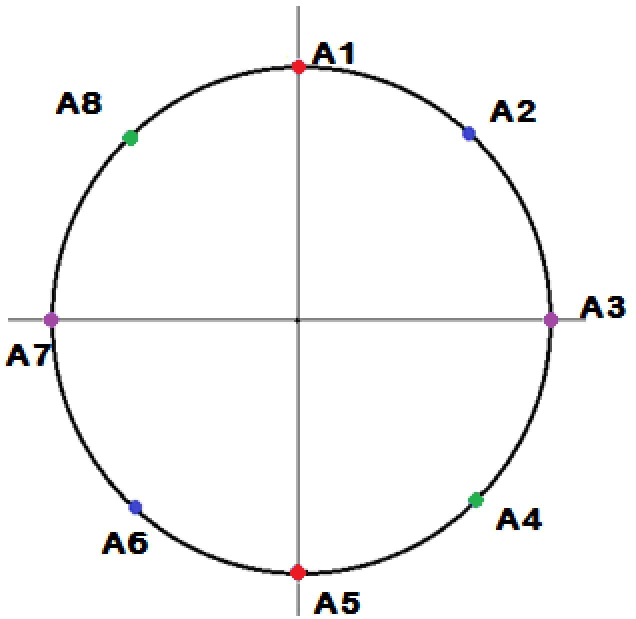
$$A_l = \begin{pmatrix} 13 & -60 \\ -20 & 93 \end{pmatrix}$$

Lifting Does Not Preserve Structure!



Next Idea: Decoupled Sampling

- Take samples of the system
- Find the DSF at each sample point
- Find a new time invariant representation
- Compute the DSF
- Compare

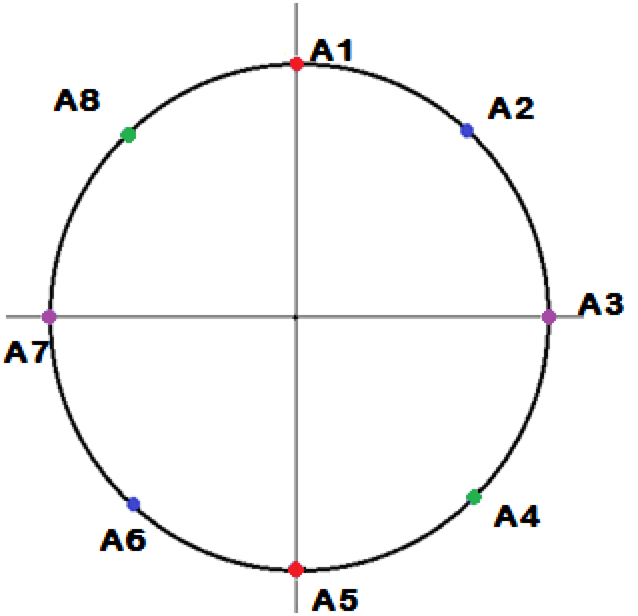


Decoupled Sampling

- How we obtain the time-invariant representation:

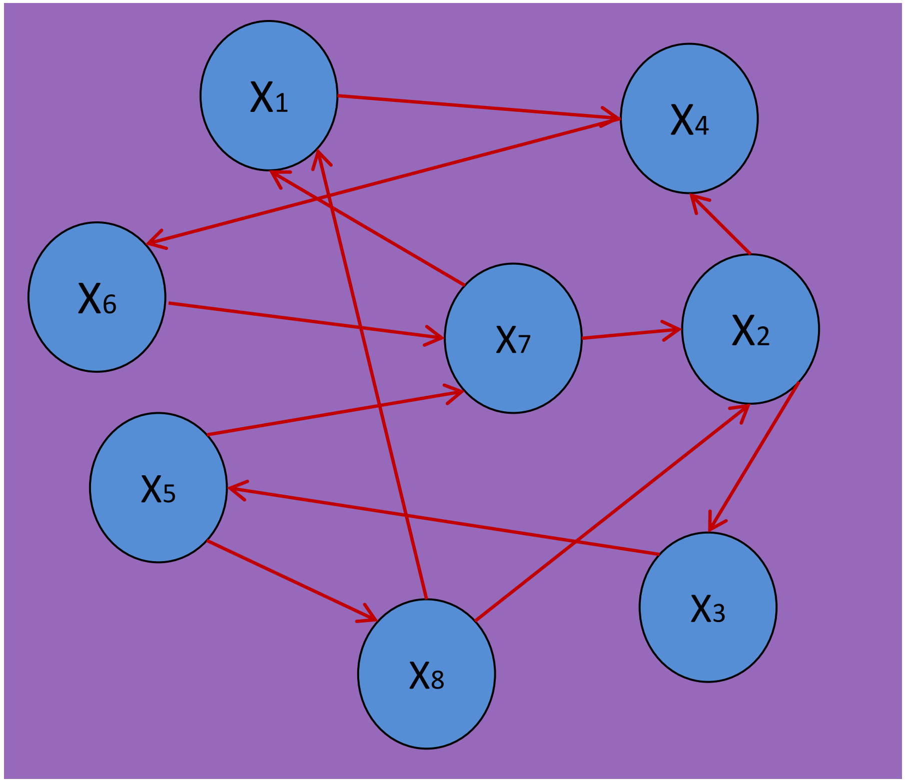
$$\begin{bmatrix} \dot{\vec{x}}_1 \\ \dot{\vec{x}}_2 \\ \dot{\vec{x}}_3 \\ \dot{\vec{x}}_4 \\ \dot{\vec{x}}_5 \\ \dot{\vec{x}}_6 \\ \dot{\vec{x}}_7 \\ \dot{\vec{x}}_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & A8 \\ A1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A7 & 0 \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \\ \vec{x}_4 \\ \vec{x}_5 \\ \vec{x}_6 \\ \vec{x}_7 \\ \vec{x}_8 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vec{u}_1 \\ \vec{u}_2 \\ \vec{u}_3 \\ \vec{u}_4 \\ \vec{u}_5 \\ \vec{u}_6 \\ \vec{u}_7 \\ \vec{u}_8 \end{bmatrix}$$

$$\begin{bmatrix} \vec{y}_1 \\ \vec{y}_2 \\ \vec{y}_3 \\ \vec{y}_4 \\ \vec{y}_5 \\ \vec{y}_6 \\ \vec{y}_7 \\ \vec{y}_8 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \\ \vec{x}_4 \\ \vec{x}_5 \\ \vec{x}_6 \\ \vec{x}_7 \\ \vec{x}_8 \end{bmatrix} \vec{H}$$



Decoupled Sampling: Results

- The structure of the system is kept
  - Contains information about the system



New A Matrix

Conclusions

- Structures of non linear systems with equilibrium points have been studied in the past
  - State Space Realization
  - Transfer Function
  - Dynamical Structure Function
- Limit cycles
- Developed a new way of representing time-varying systems with a time-invariant structure
  - Keeps structure

Future Work

- Developing Network Reconstruction algorithms to find the oscillating structure from data using the decoupled sampling technique
- Explore time varying representations of structure

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References

[1] "Model reduction of periodic systems: a lifting approach," *Automatica*, vol. 41, no. 6, p. 1088, Jul. 2003.

[2] "Analysis and Synthesis of Controllers for the Classes of Slowly Varying, Periodic and Multirate Systems," Ph.D Dissertation, Dept. AA, M.I.T., MA, 1991.

[3] S. Warnick, "Dynamical Structure Functions for the Reverse Engineering of LTI Networks," *46th IEEE Conference on Decision and Contro*, New Orleans, LA, 2007, pp. 1518-1519.