The Information Cost of Realization

Philip E. Paré

M.S. Computer Science

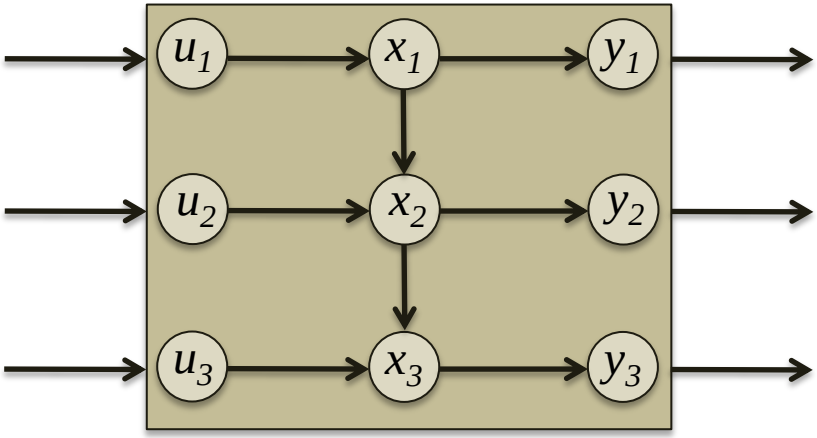
Advisor: Sean Warnick

Information & Decision Algorithms Laboratories

Presentation Outline

- Systems and Structure
- Identifying System Structure from Data
- Theorem: Information Cost of Realization
- Generalized Example
- Conclusions/Future Work

Two System Representations

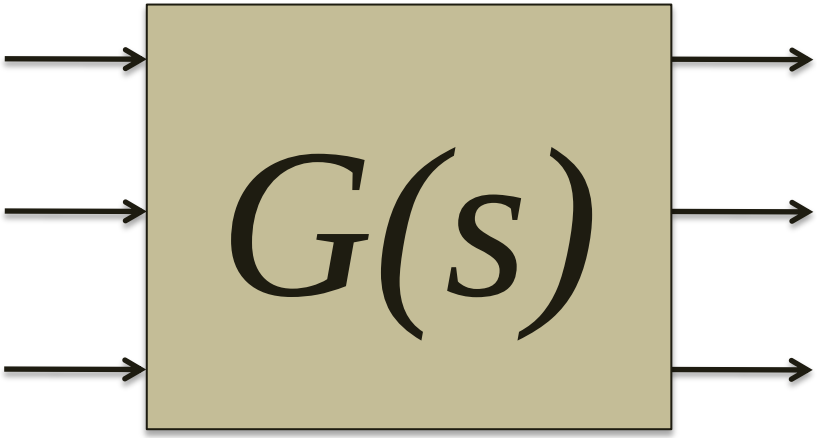


State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

What is System Structure?

Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

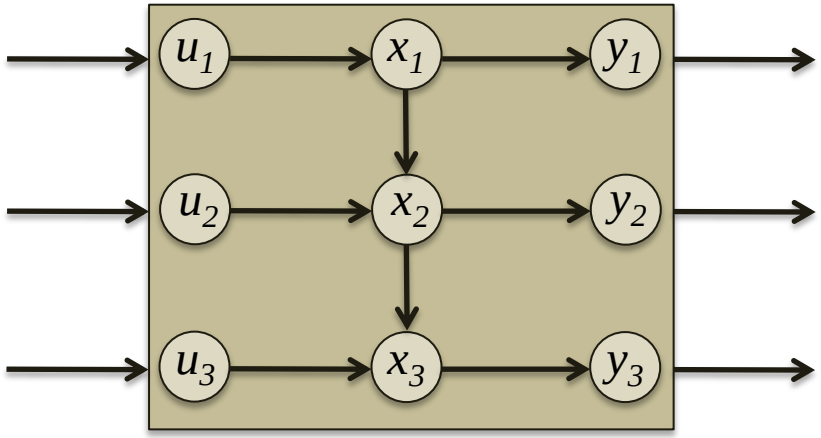
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

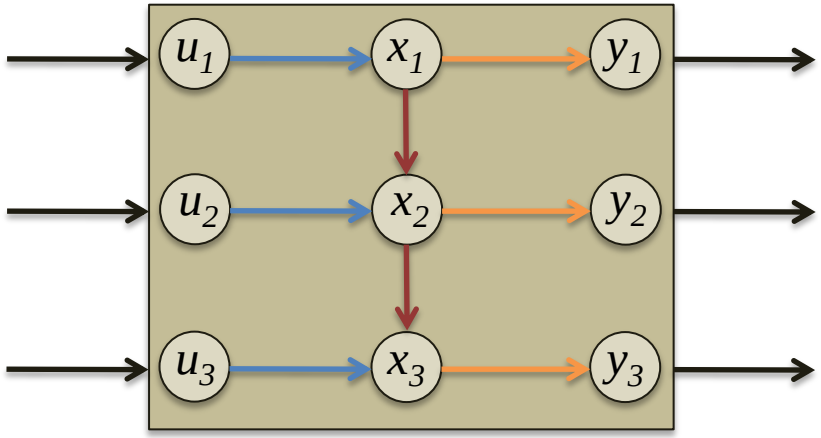
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

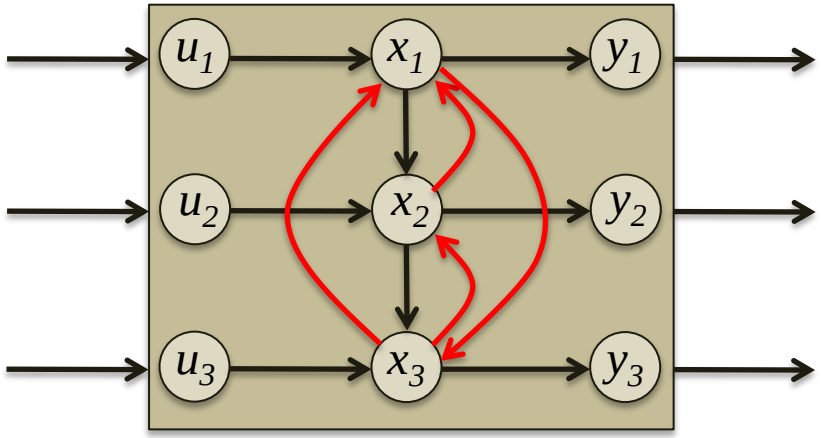
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

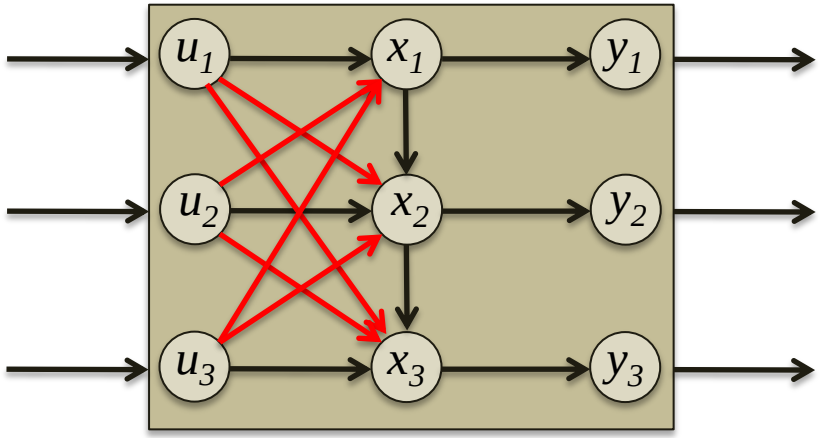
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

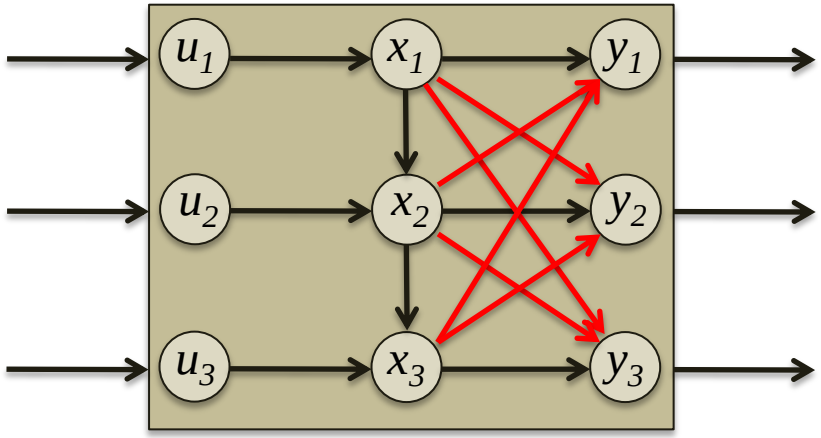
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

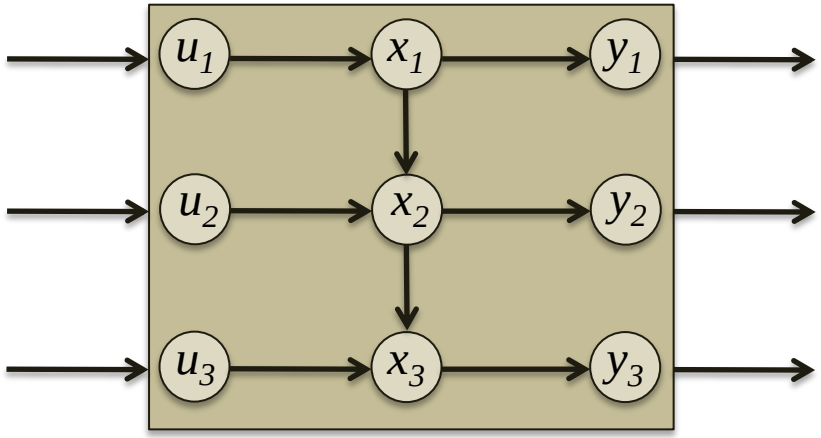
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

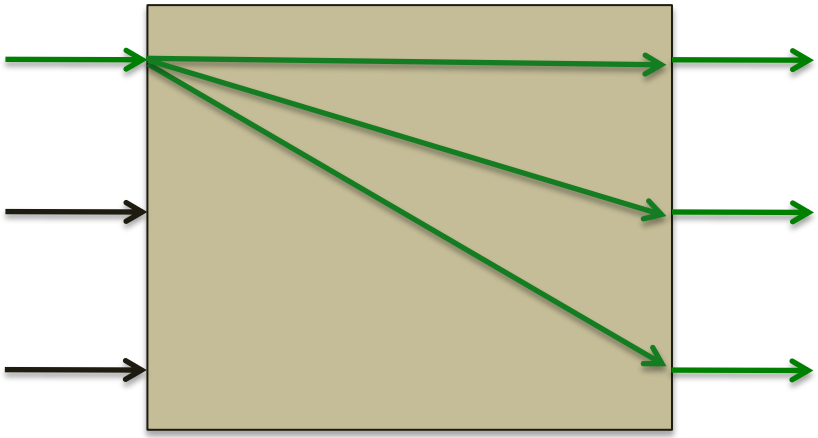
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

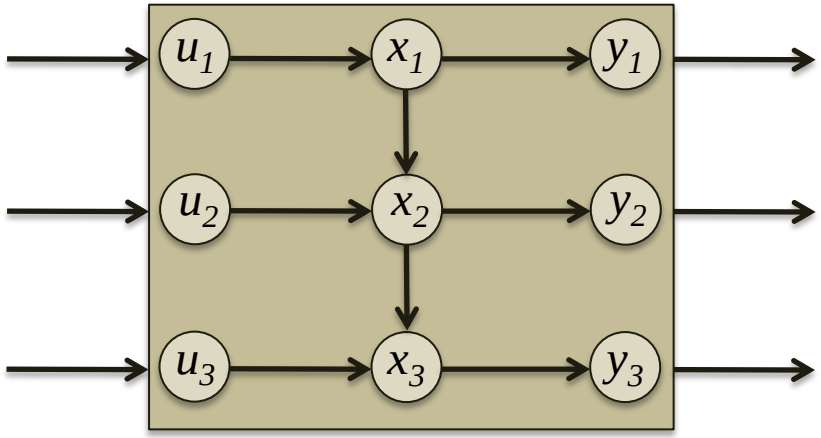
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

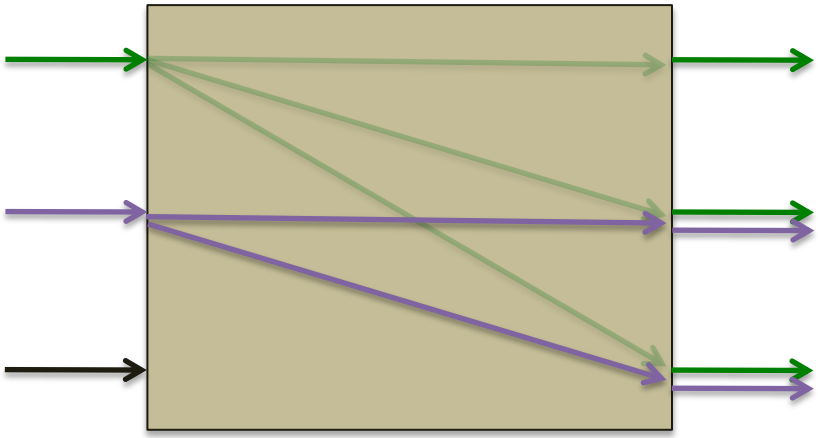
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

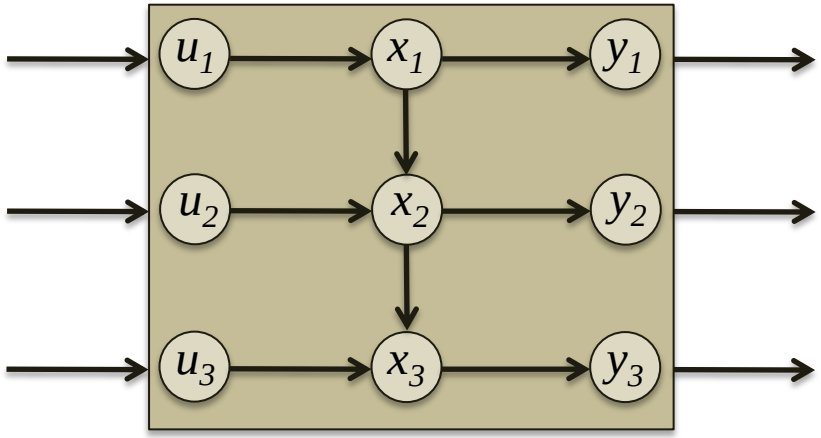
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

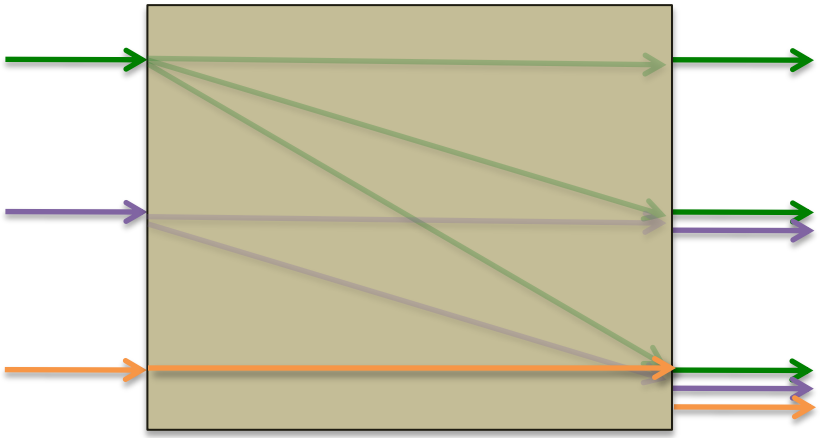
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

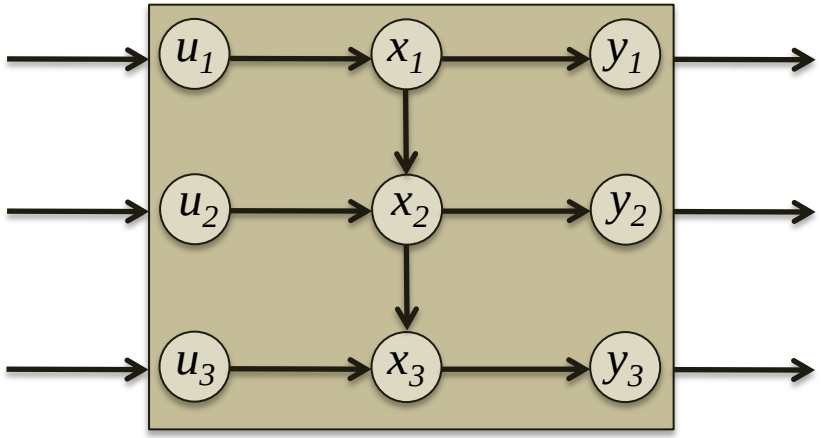
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

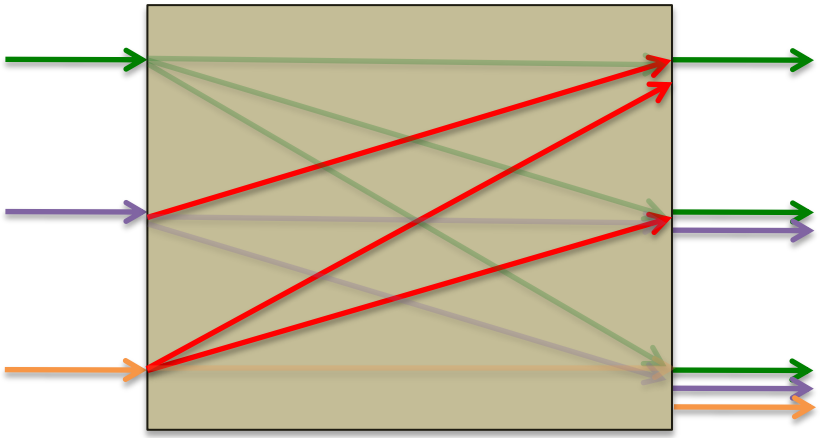
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

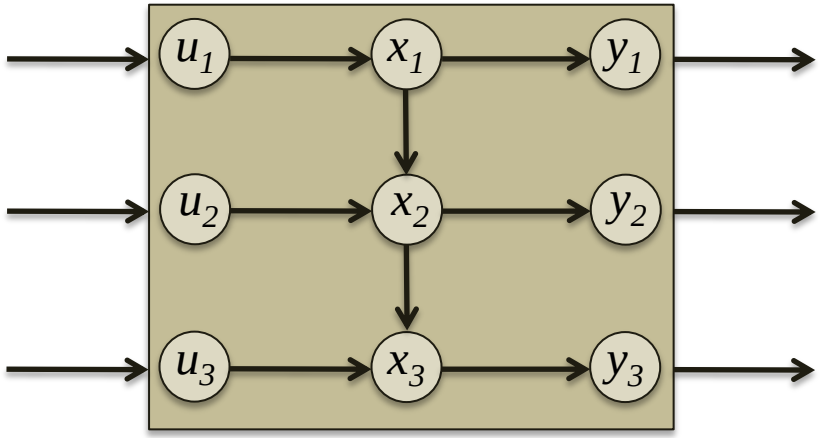
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

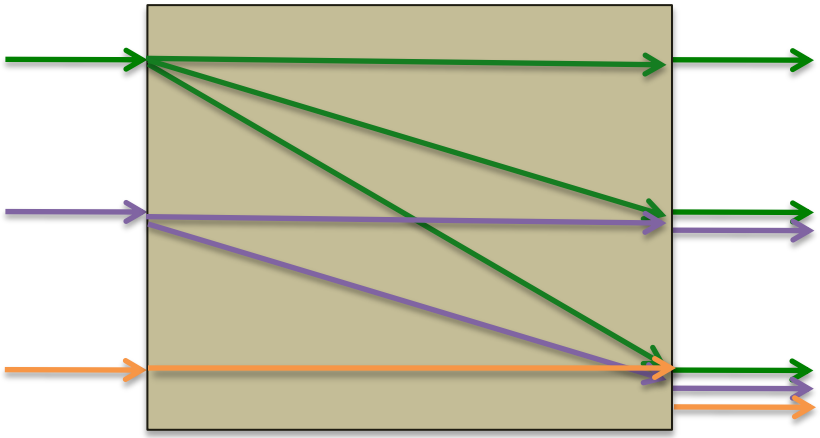
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

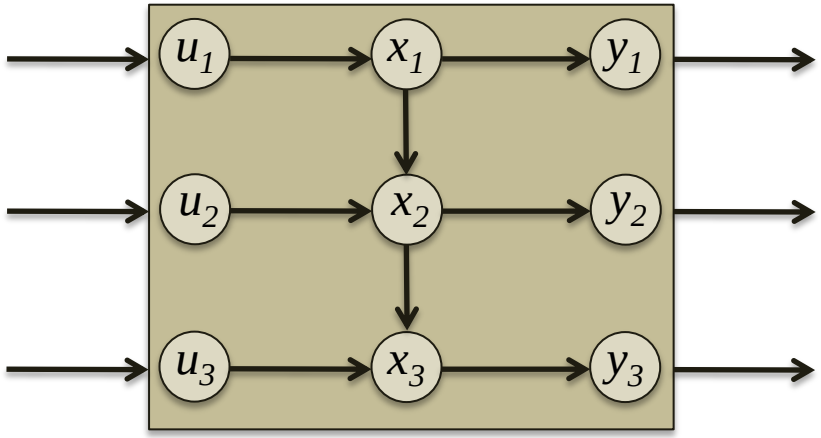
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



Transfer Function (G)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

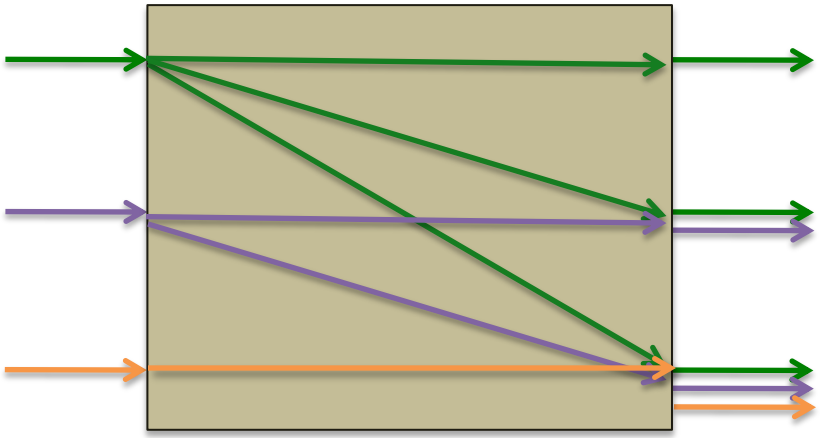
State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

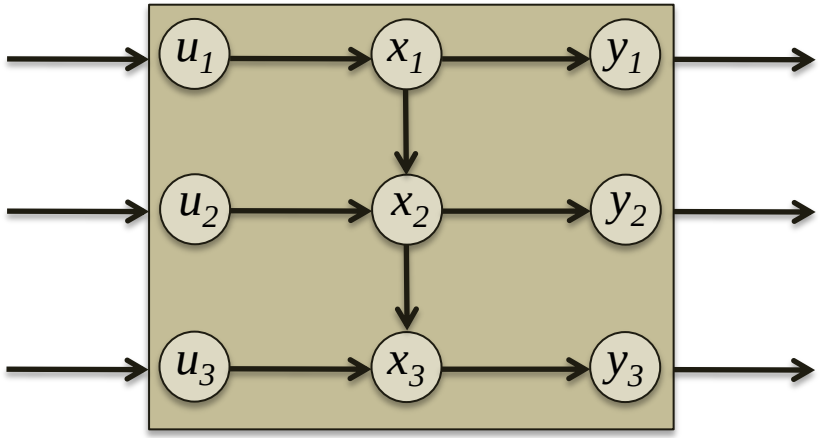
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Two System Representations

Sparsity Pattern of the Transfer Function



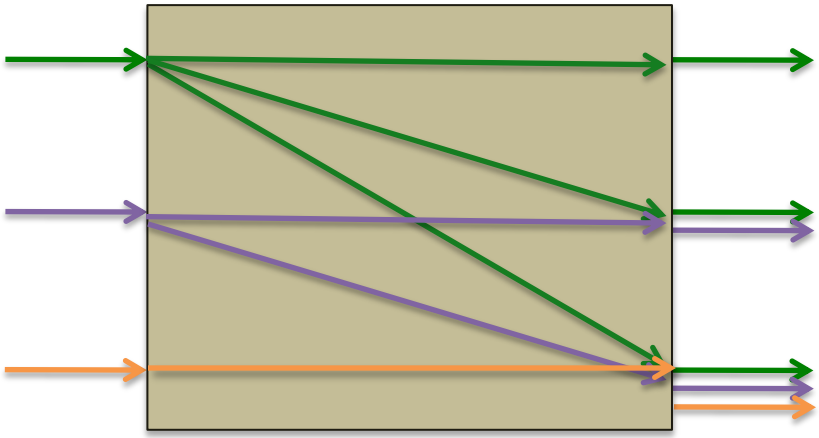
Complete Computational Structure



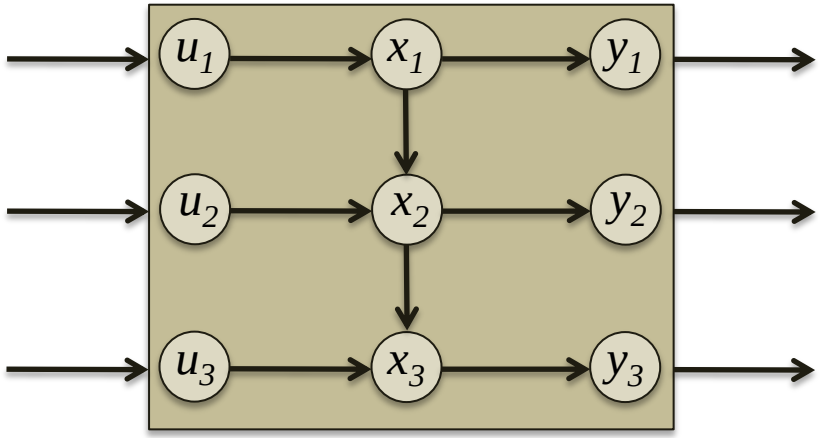
- These “structures” are weaker/stronger versions of each other
  - Weak: closed-loop paths relating manifest variables (inputs/outputs)
  - Strong: direct interaction among physical (fundamental) components

Two System Representations

Sparsity Pattern of the Transfer Function



Complete Computational Structure



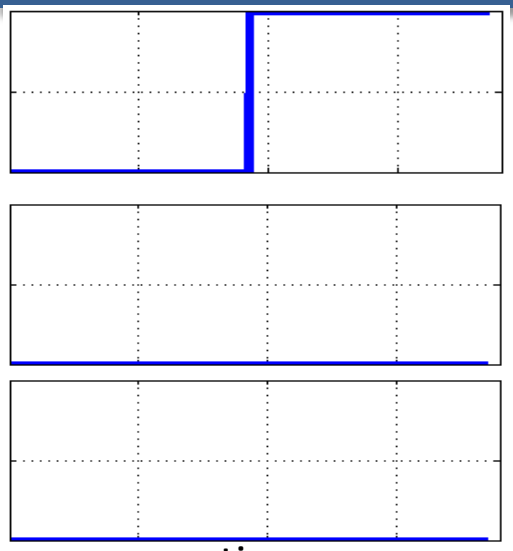
- These "structures" are weaker/stronger versions of each other
  - Weak: closed-loop paths relating manifest variables (inputs/outputs)
  - Strong: direct interaction among physical (fundamental) components
- Which kind of "structure" do network reconstruction algorithms try to find?

Two System Representations

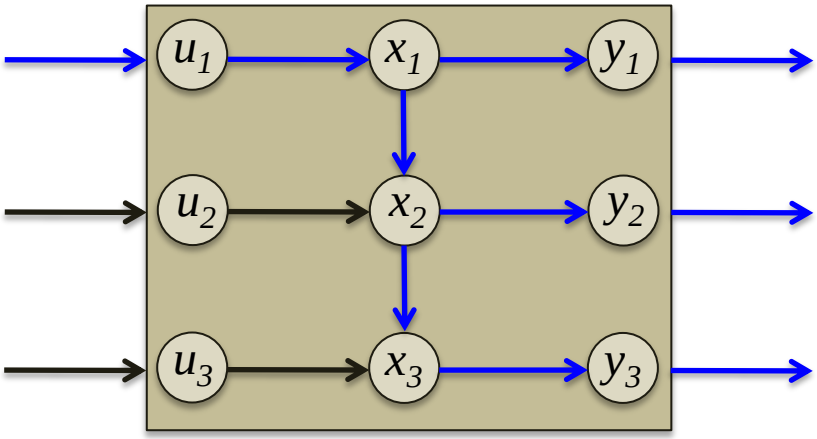
Two System Representations

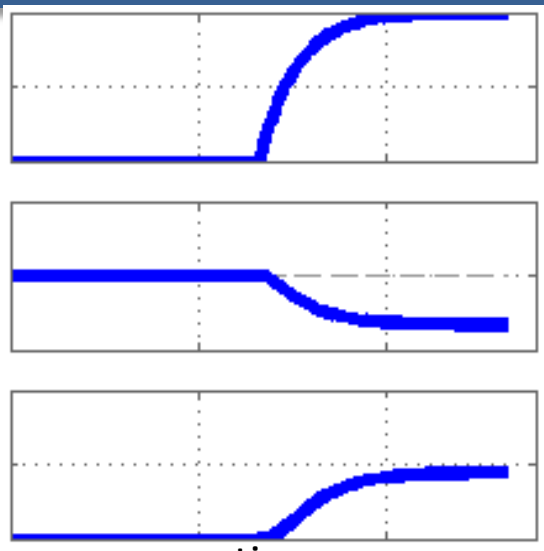
What do system identification algorithms do?

Identifying System Structure from Data



time





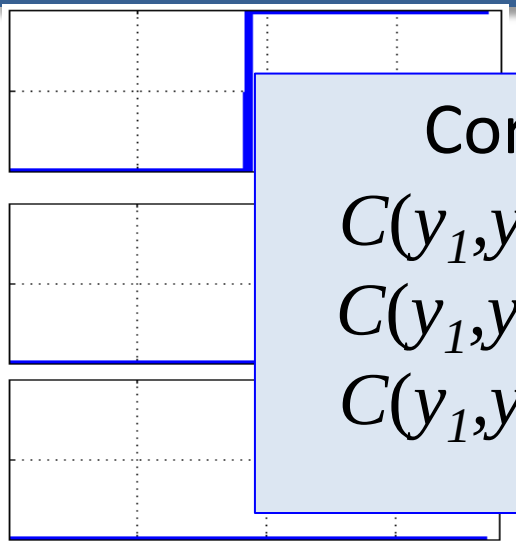
time

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data

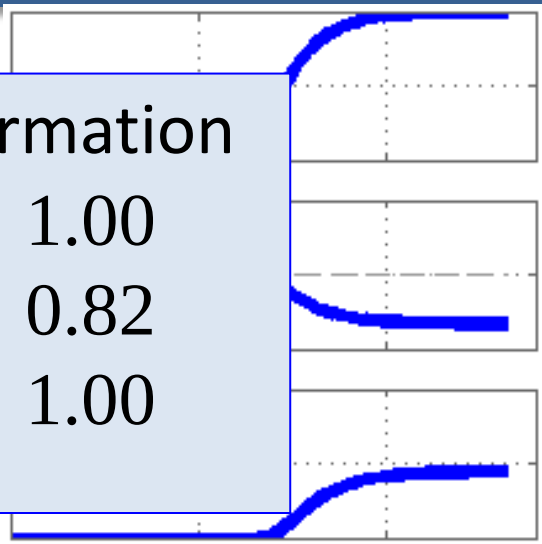


Correlations

$$C(y_1,y_1) = 1.0000$$
$$C(y_1,y_2) = -0.9898$$
$$C(y_1,y_3) = 0.9621$$

Mutual Information

$$I(y_1,y_1) = 1.00$$
$$I(y_1,y_2) = 0.82$$
$$I(y_1,y_3) = 1.00$$



time

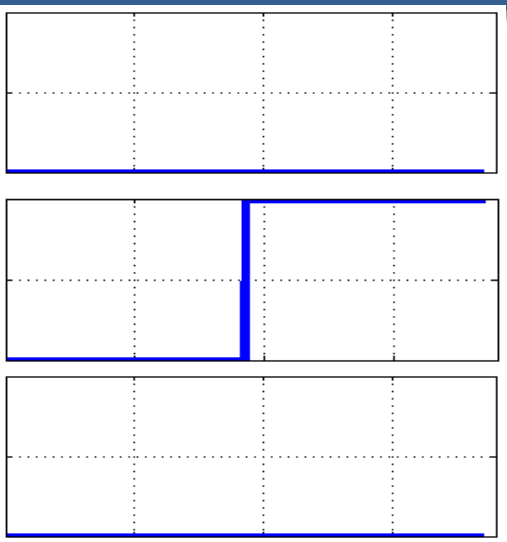
time

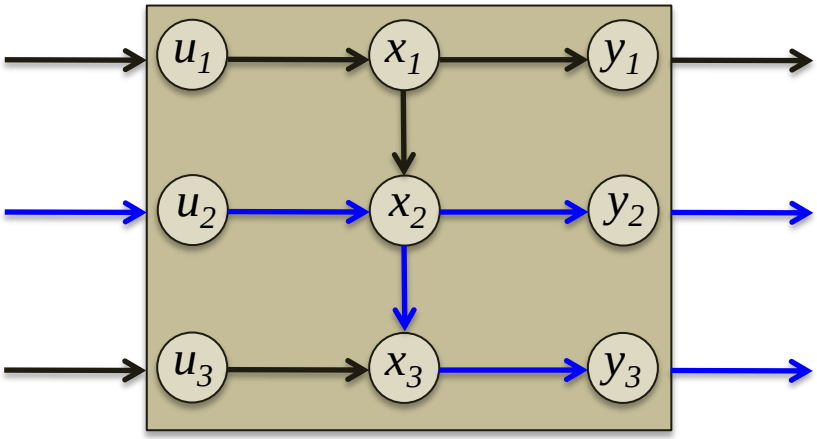
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

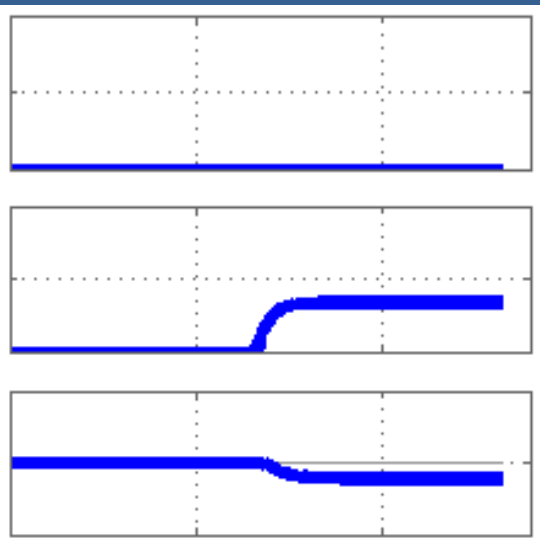
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data







time

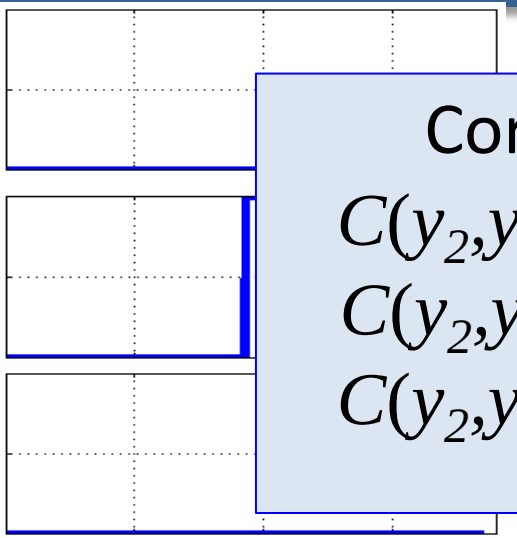
time

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data

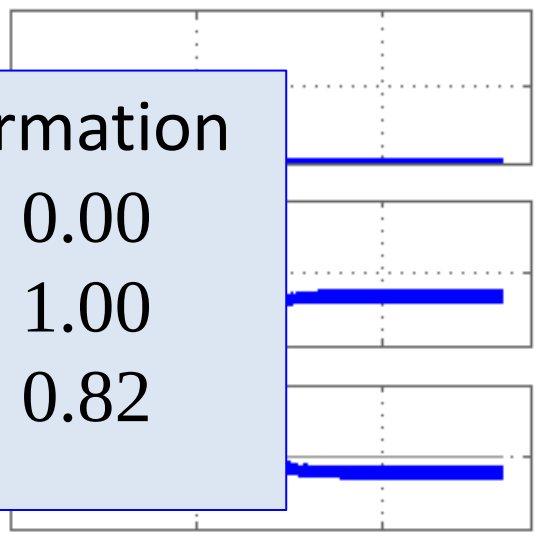


Correlations

$$C(y_2,y_1) = -0.0000$$
$$C(y_2,y_2) = 1.0000$$
$$C(y_2,y_3) = -0.9407$$

Mutual Information

$$I(y_2,y_1) = 0.00$$
$$I(y_2,y_2) = 1.00$$
$$I(y_2,y_3) = 0.82$$



time

time

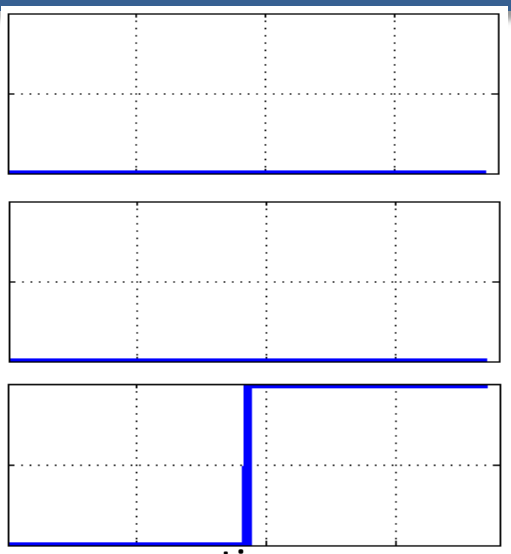
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

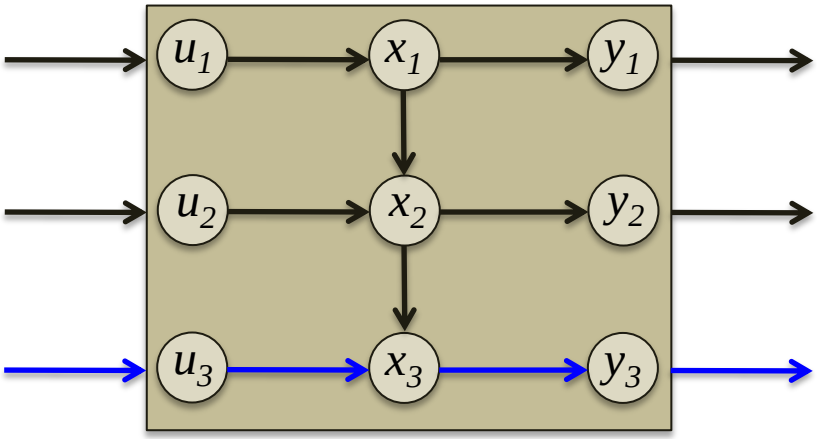
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

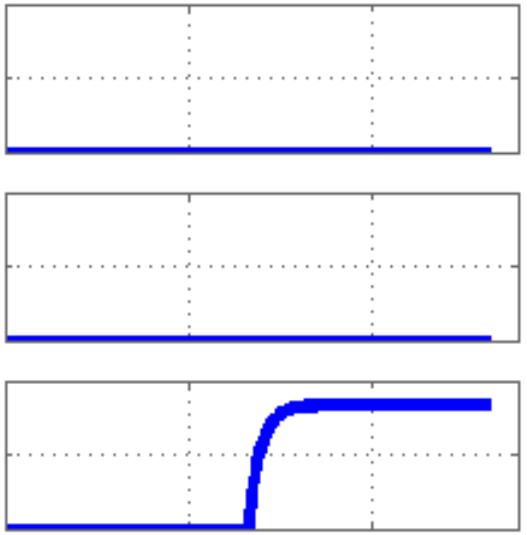
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

IDeA Labs
Information & Decision Algorithms Laboratories

Identifying System Structure from Data





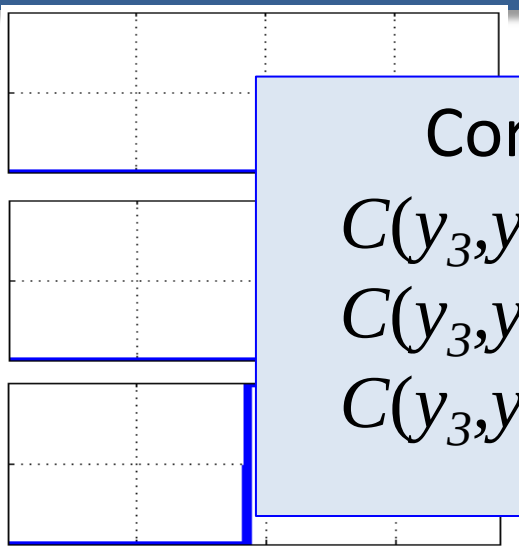


$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data

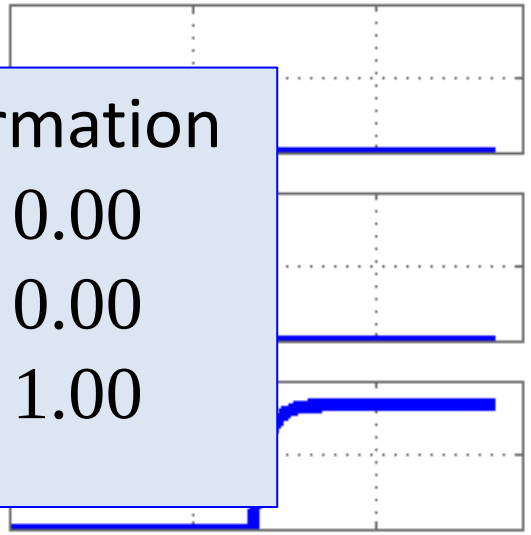


Correlations

$$C(y_3,y_1) = 0.0000$$
$$C(y_3,y_2) = 0.0000$$
$$C(y_3,y_3) = 1.0000$$

Mutual Information

$$I(y_3,y_1) = 0.00$$
$$I(y_3,y_2) = 0.00$$
$$I(y_3,y_3) = 1.00$$



$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data

Correlations

$$\begin{bmatrix} 1.000 & 0.0000 & 0.0000 \\ -0.9898 & 1.000 & 0.0000 \\ 0.9621 & -0.9407 & 1.000 \end{bmatrix}$$

Mutual Information

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.82 & 1.00 & 0.00 \\ 1.00 & 1.82 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

IDeA Labs
Information & Decision Algorithms Laboratories

Identifying System Structure from Data

Correlations

$$\begin{bmatrix} 1.000 & 0.0000 & 0.0000 \\ -0.9898 & 1.000 & 0.0000 \\ 0.9621 & -0.9407 & 1.000 \end{bmatrix}$$

Mutual Information

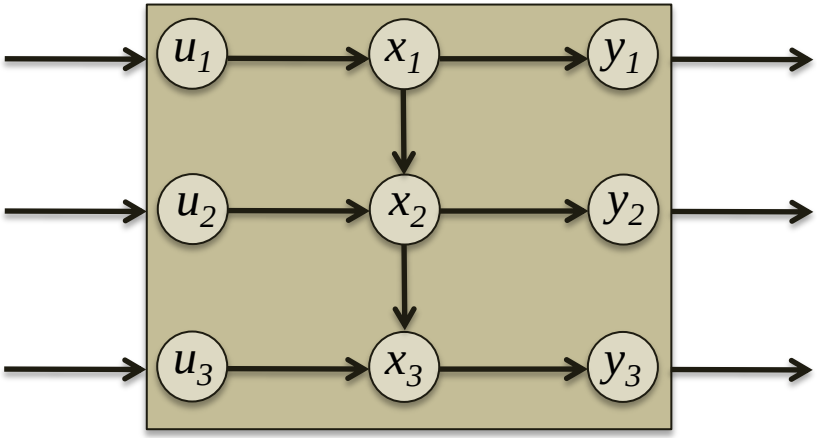
$$\begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.82 & 1.00 & 0.00 \\ 1.00 & 1.82 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



Transfer Function (G)

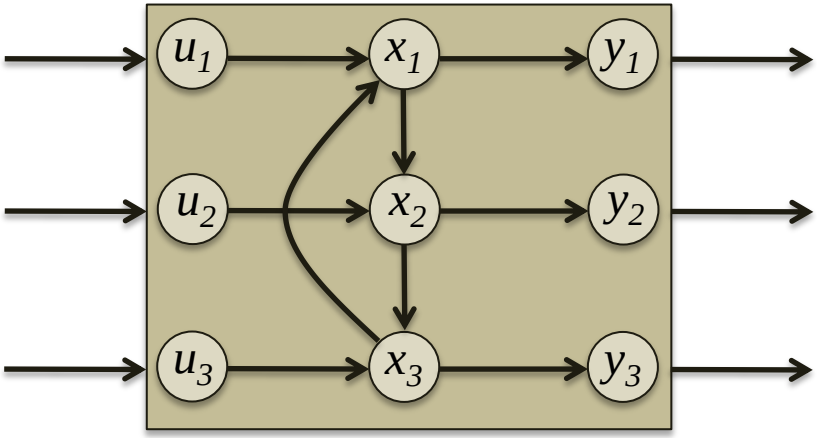
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s+3} & 0 \\ \frac{4}{(s+1)(s+3)^2} & \frac{-2}{(s+3)^2} & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



Transfer Function (G)

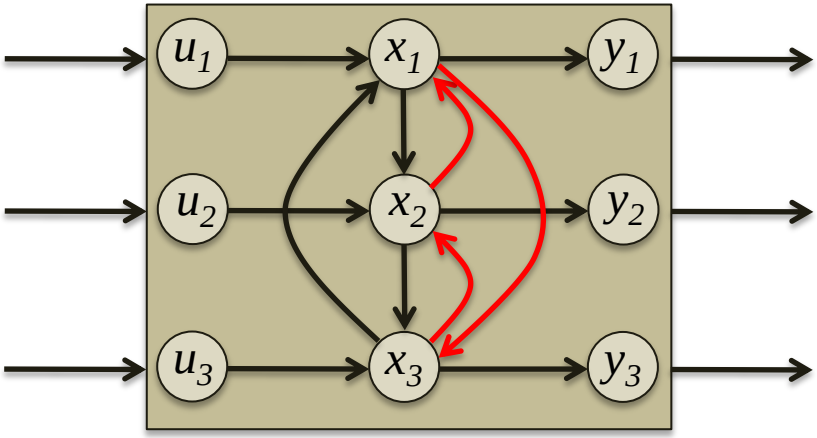
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

State Space Realization (A,B,C,D)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



Transfer Function (G)

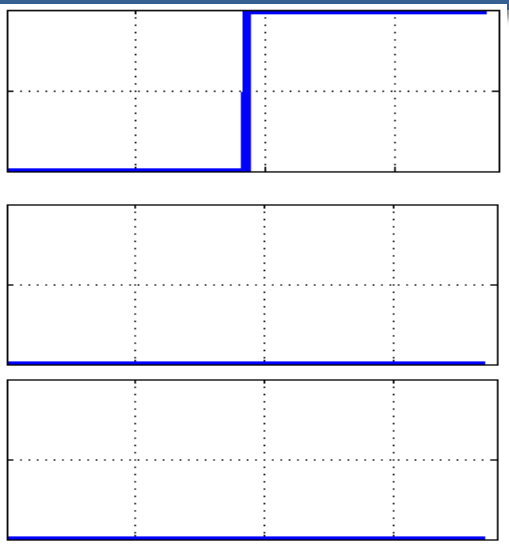
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

State Space Realization (A,B,C,D)

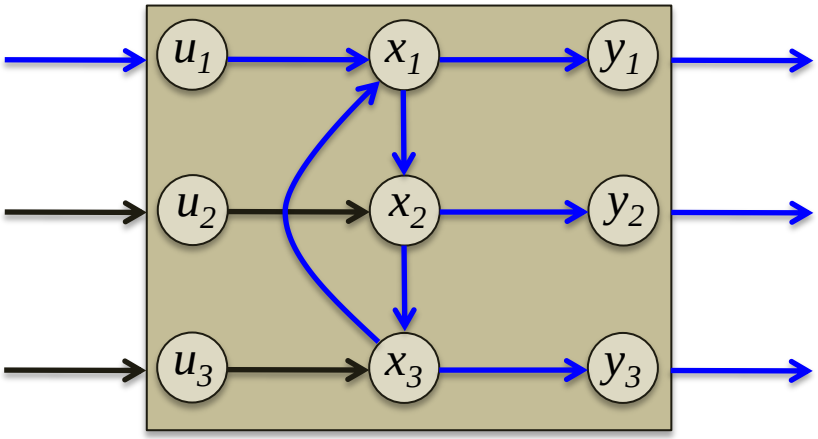
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

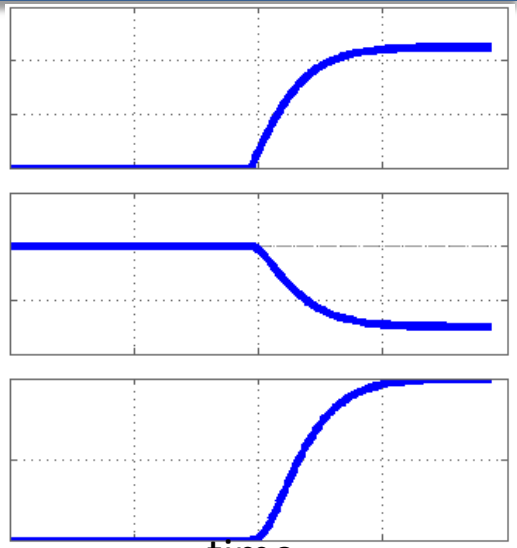
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



time





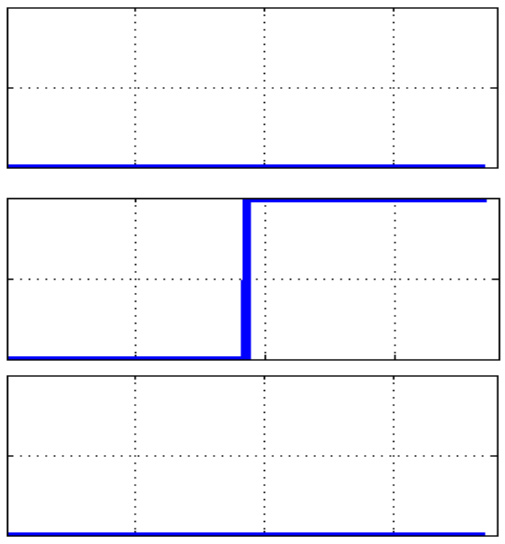
time

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

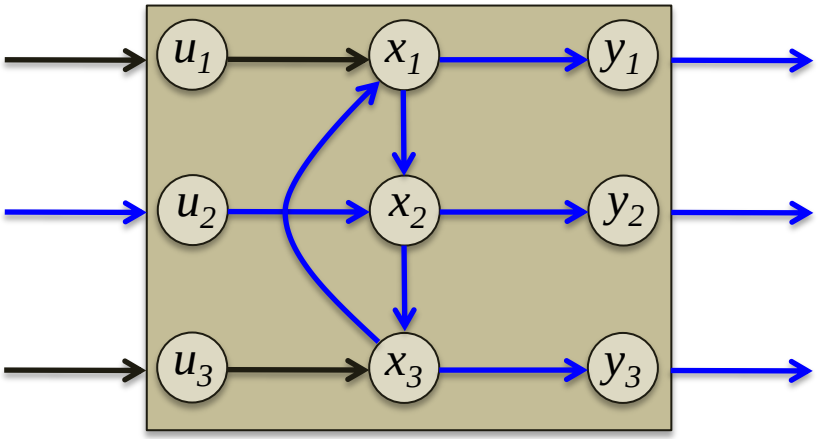
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

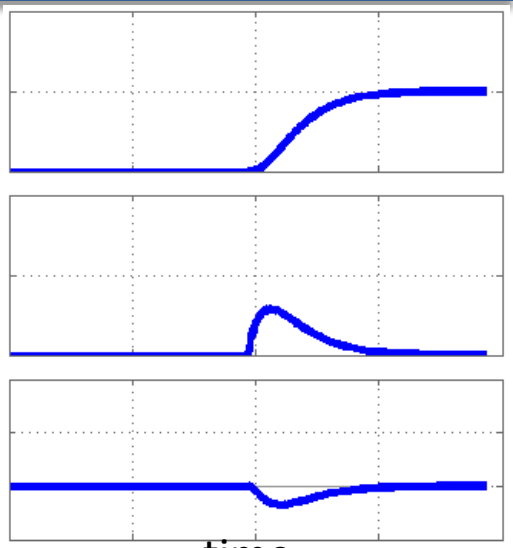
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



time





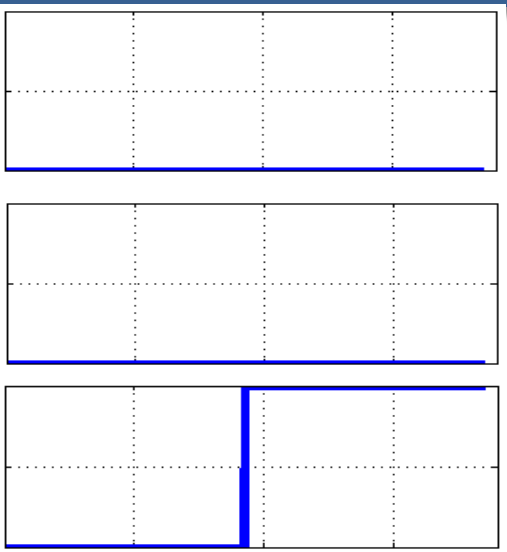
time

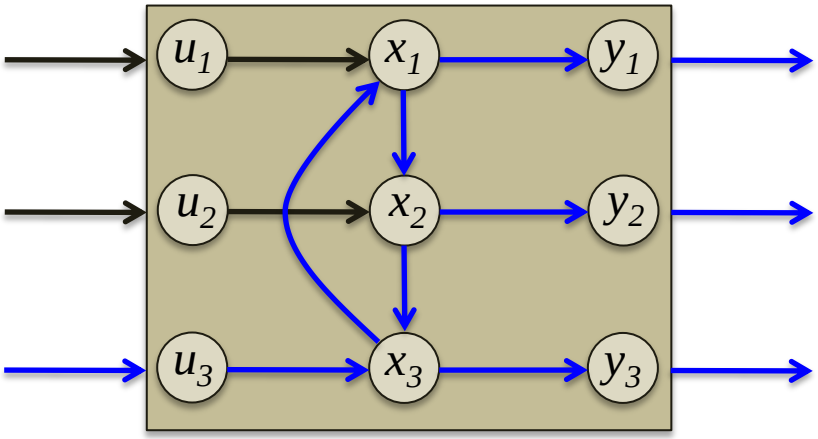
$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

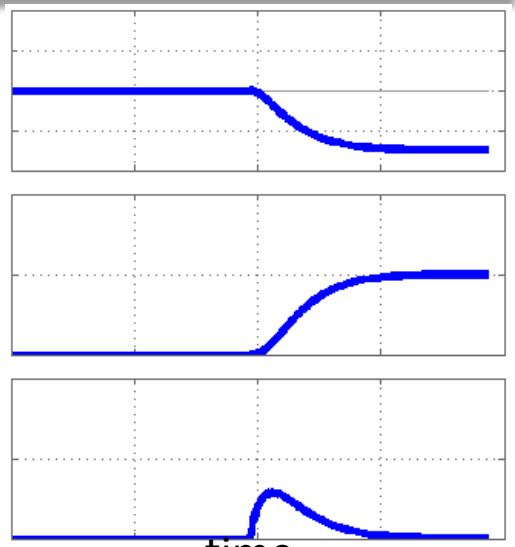
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data







time

time

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

IDeA Labs
Information & Decision Algorithms Laboratories

Identifying System Structure from Data

Correlations

$$\begin{bmatrix} 1.000 & -0.8391 & 0.7852 \\ -0.9953 & 1.000 & -0.8391 \\ 0.9815 & -0.9296 & 1.000 \end{bmatrix}$$

Mutual Information

$$\begin{bmatrix} 1.00 & 0.71 & 0.77 \\ 0.79 & 1.00 & 0.71 \\ 1.00 & 0.73 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

IDeA Labs
Information & Decision Algorithms Laboratories

Identifying System Structure from Data

Correlations

$$\begin{bmatrix} 1.000 & -0.8391 & 0.7852 \\ -0.9953 & 1.000 & -0.8391 \\ 0.9815 & -0.9296 & 1.000 \end{bmatrix}$$

Mutual Information

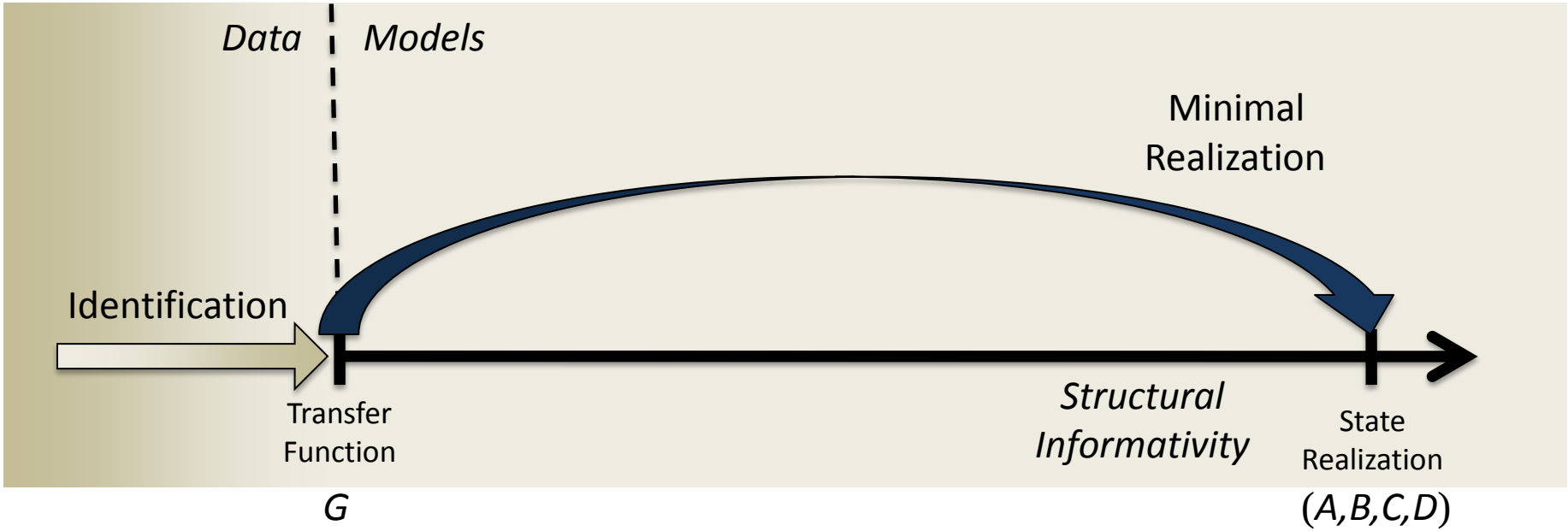
$$\begin{bmatrix} 1.00 & 0.71 & 0.77 \\ 0.79 & 1.00 & 0.71 \\ 1.00 & 0.73 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \frac{(s+3)^2}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} & \frac{-(s+3)}{(s+1)^2(s+4)} \\ \frac{-2(s+3)}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} & \frac{2}{(s+1)^2(s+4)} \\ \frac{4}{(s+1)^2(s+4)} & \frac{-2s}{(s+1)^2(s+4)} & \frac{s(s+3)}{(s+1)^2(s+4)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -2 & -3 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

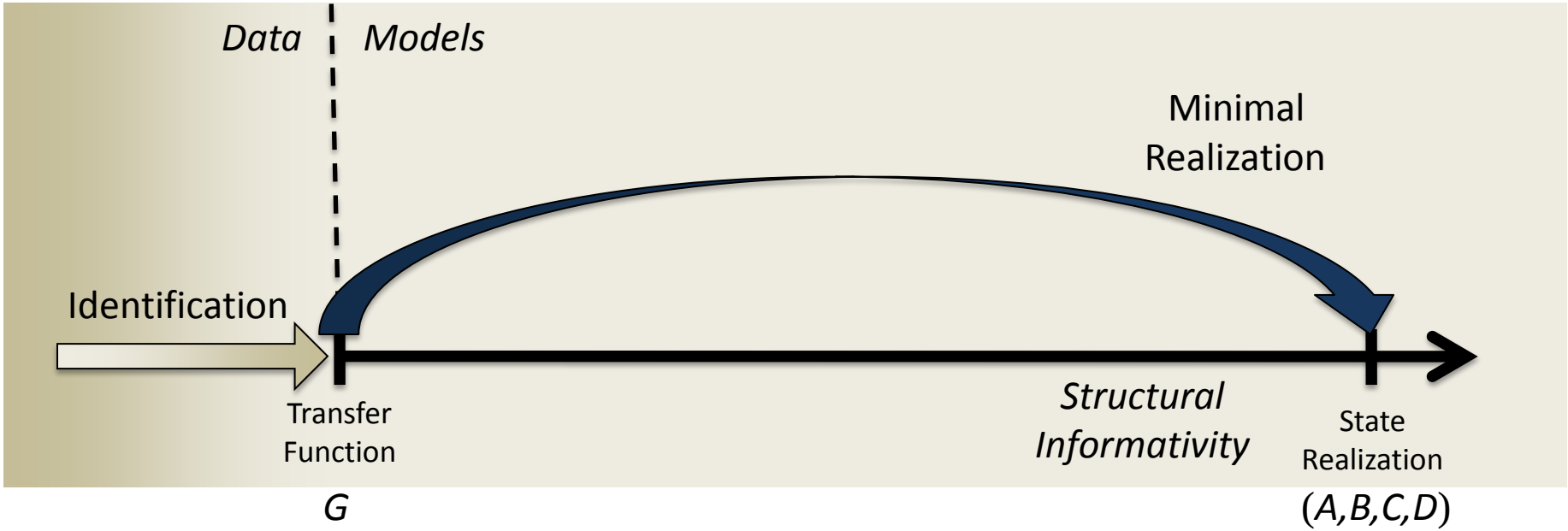
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Identifying System Structure from Data



- These methods find the black box, or Transfer Function
- Do not find the internal sparsity pattern of the underlying State Space Realization

Identifying System Structure from Data



- How do network reconstruction methods find the true (A,B,C,D) from data when many (A,B,C,D) generate the same G ?
- Assume full state measurements i.e. $C = I$, $D = 0$.

Theorem: Information Cost of Realization

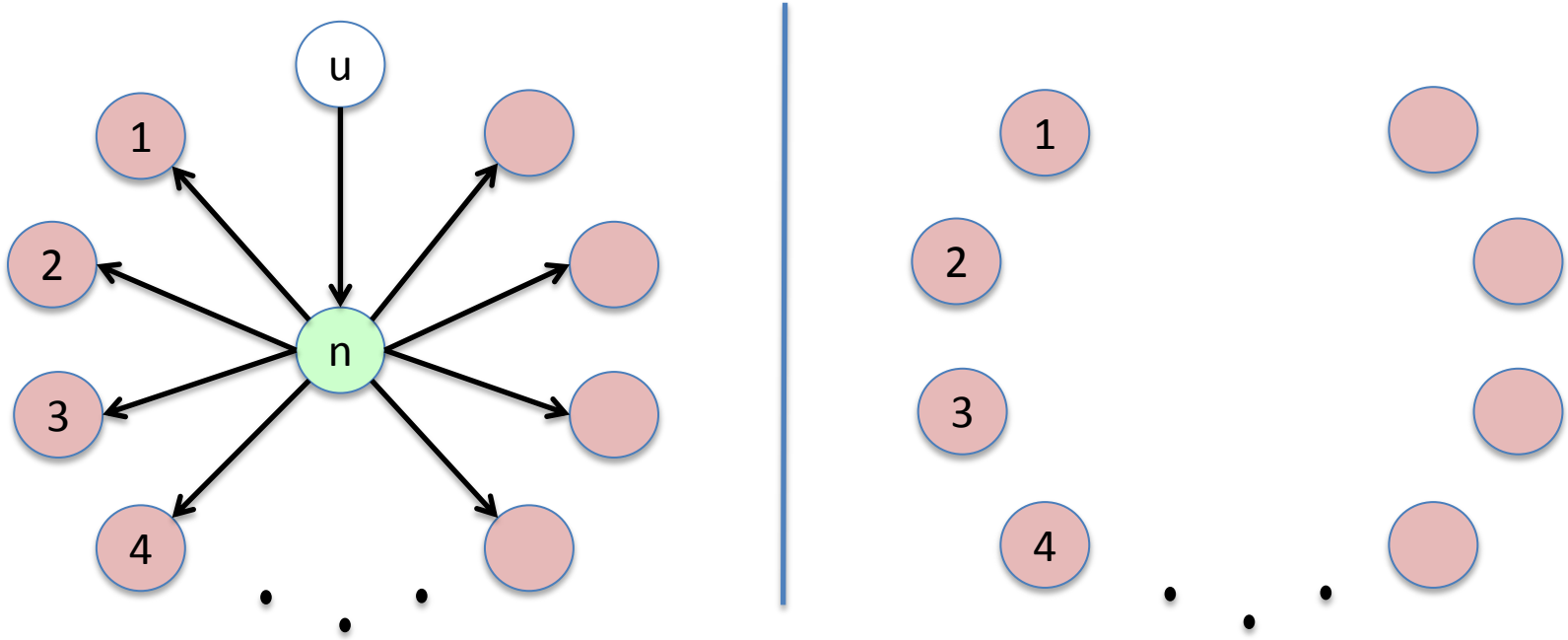
- Theorem: If *G* is fixed and *y* is a subset of *x*, there exists a unique *(A,B)* if and only if *C=I*.

Theorem: Information Cost of Realization

- Theorem: If *G* is fixed and *y* is a subset of *x*, there exists a unique *(A,B)* if and only if *C=I*.

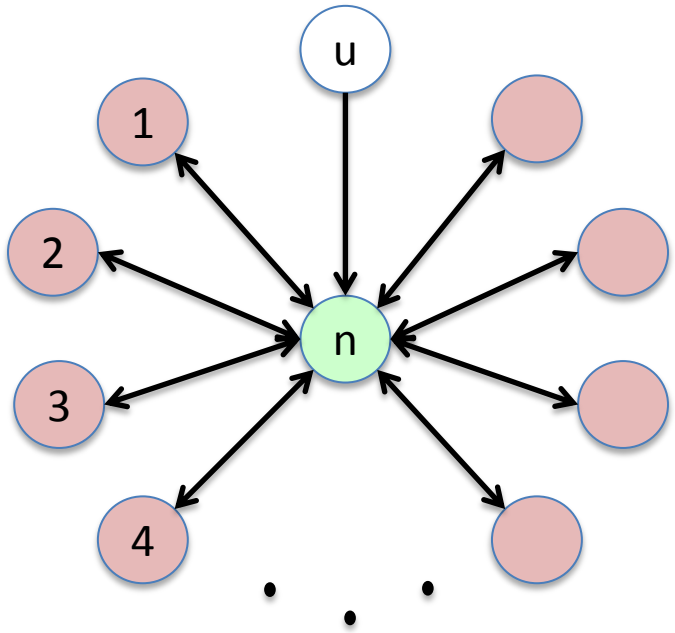
So, the ONLY way to recover (A,B,C,D) from G is to assume full-state feedback.

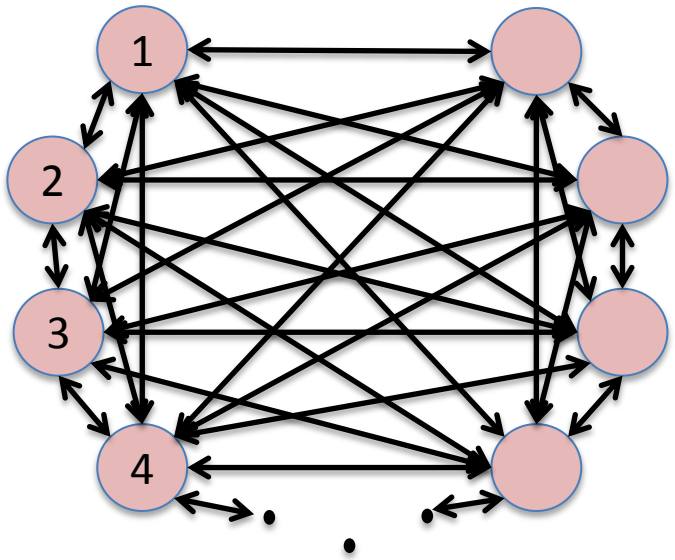
Generalized Example



Both realizations are minimal,
recover completely different structures

Generalized Example





Both realizations are minimal,
recover completely different structures

Conclusions/ Future Work

- Information cost of state realization
  - You need to know that you are measuring ALL of the system states
- Future work will consider situations where your measurements are not a subset of the system states
- Thank you

Theorem Proof

- We will show by contradiction that if C=I then there is a unique (A,B) that realizes it.
- Assume C= I and that G(s) is realized by (A_1,B_1) and (A_2,B_2) where $A_1 \neq A_2$ and $B_1 \neq B_2$. Since C= I, $G(s) = (sI - A_1)^{-1} B_1 = (sI - A_2)^{-1} B_2$.
- Clearly $B_1 = M B_2$ where $M = (sI - A_1)(sI - A_2)^{-1}$. Note that since both B_1 and B_2 have no s terms in them, there are no s terms in M. It follows that
- $(sI - A_1) = M (sI - A_2) = sM - MA_2$
- Bringing everything to one side and combining like terms gives, $s(I - M) + (MA_2 - A_1) = 0$
- Since the second term has no s in it, in order for this equality to hold both terms must equal zero. The first term equaling zero gives that M = I. This combined with the second term gives that $A_1 = A_2$. Inserting M=I gives that $B_1 = B_2$. This is a contradiction so there is only one set of matrices (A,B) that realizes G(s).