Necessary and Sufficient Conditions on State Transformations that Preserve the Causal Structure of LTI Dynamical Networks

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IDeA Labs
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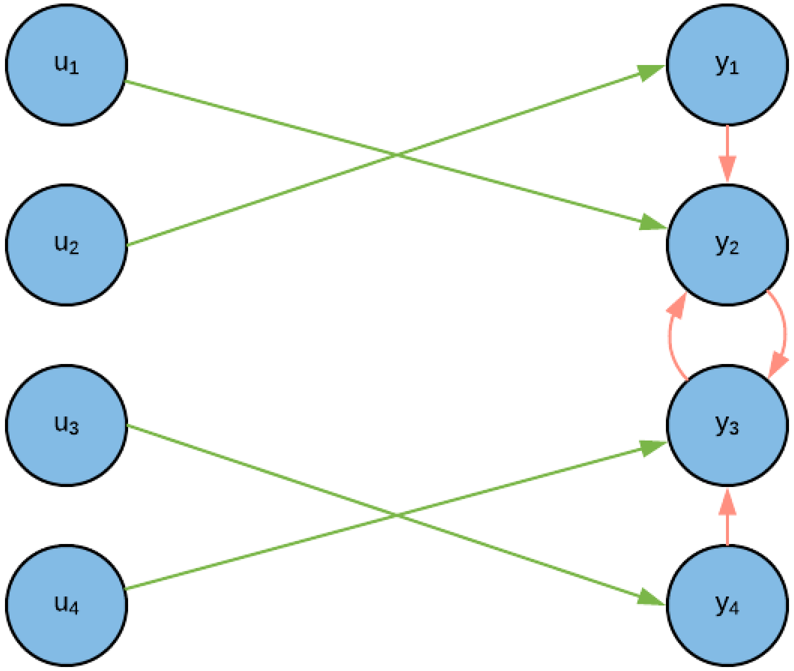
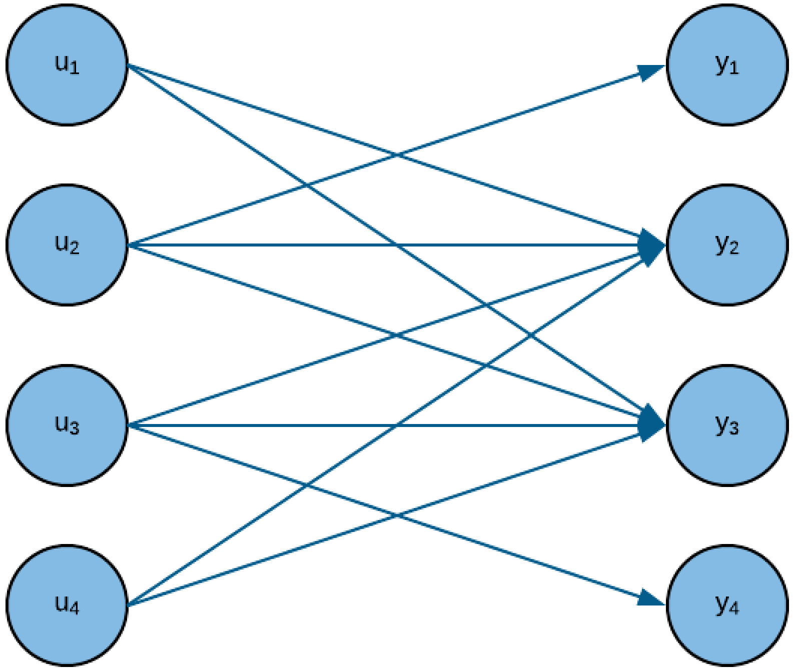
Contents

Background

Dynamical Network Function

Algebraic Relationship (DNF -> TF)





$$Y(s) = W(s)Y(s) + V(s)U(s)$$

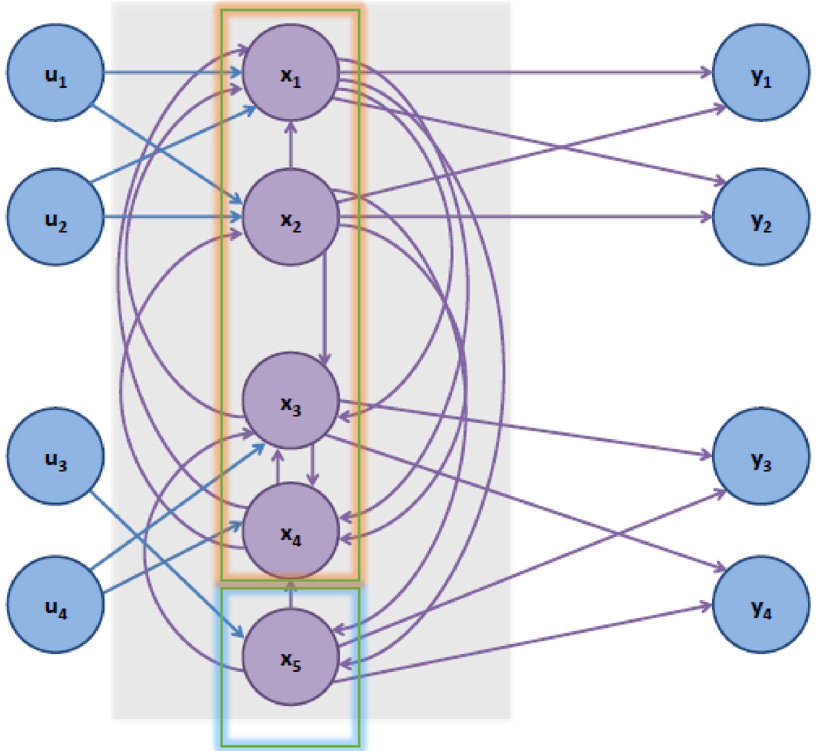
Dynamical Network Function

$$Y(s) = G(s)U(s)$$

Transfer Function

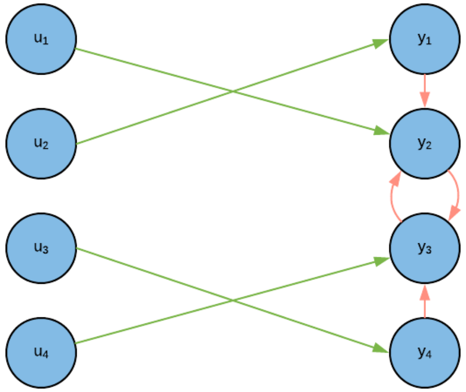
$$Y(s) = (I - W(s))^{-1}V(s)U(s)$$

Algebraic Relationship (SSR -> DNF)



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State Space Model

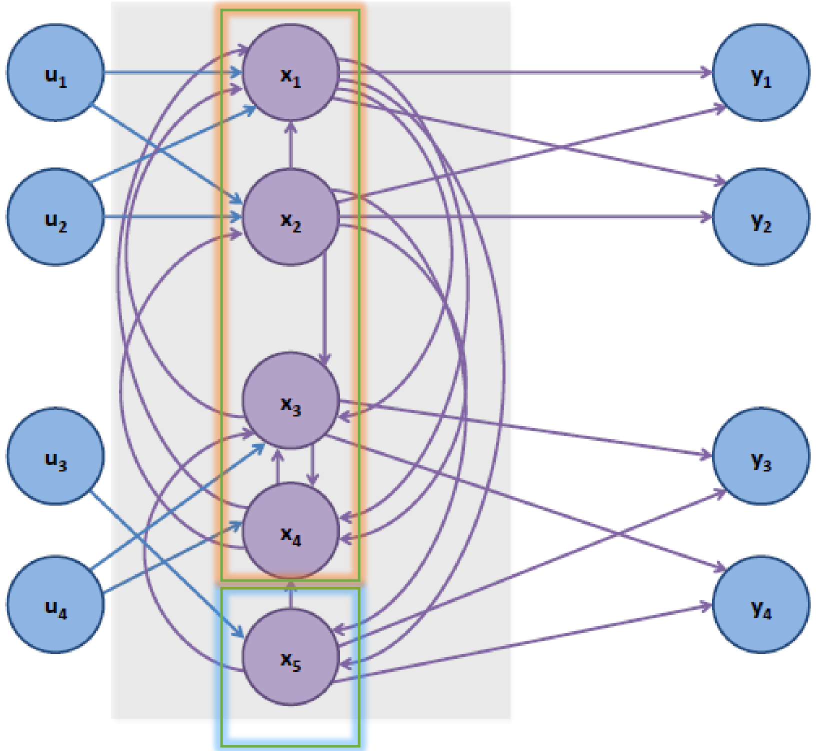


Dynamical Network Function

$$Y(s) = W(s)Y(s) + V(s)U(s)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$
$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du$$

Algebraic Relationship (SSR -> DNF)



$$\dot{x} = Ax + Bu$$
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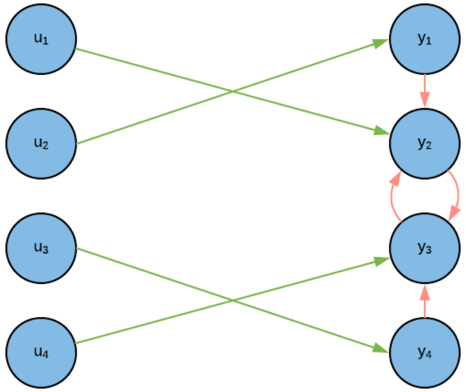
State Space Model

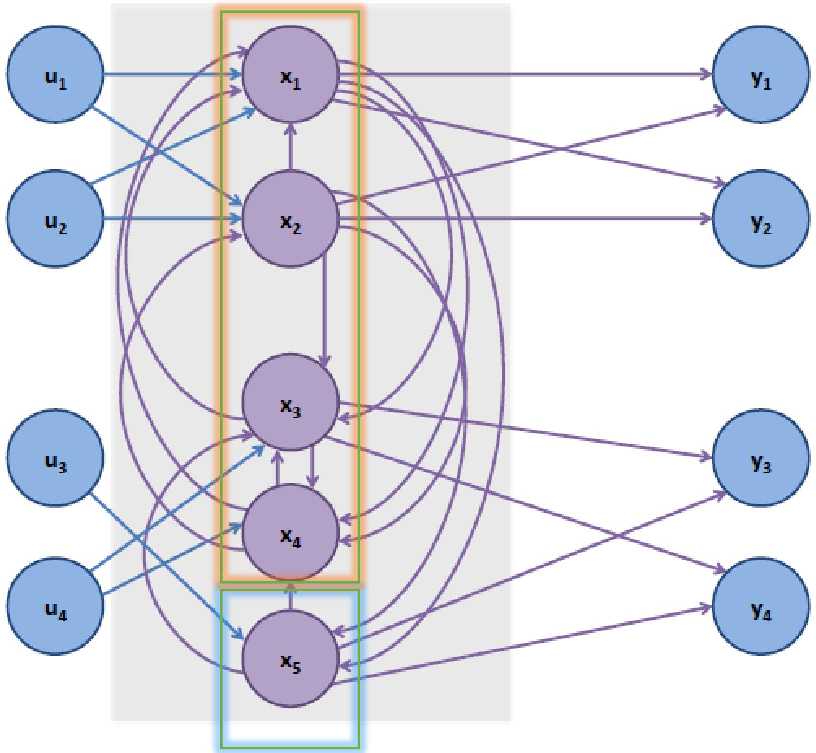


Dynamical Network Function

$$Y(s) = W(s)Y(s) + V(s)U(s)$$

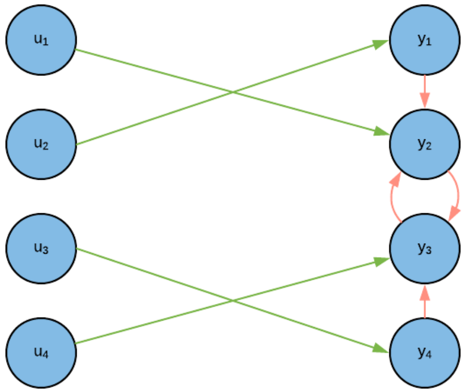
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$
$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du$$

Algebraic Relationship (SSR -> DNF)



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State Space Model



Dynamical Network Function

$$Y(s) = W(s)Y(s) + V(s)U(s)$$

$$sY = (A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})Y + (A_{12}(sI - A_{22})^{-1}B_2 + B_1)U$$

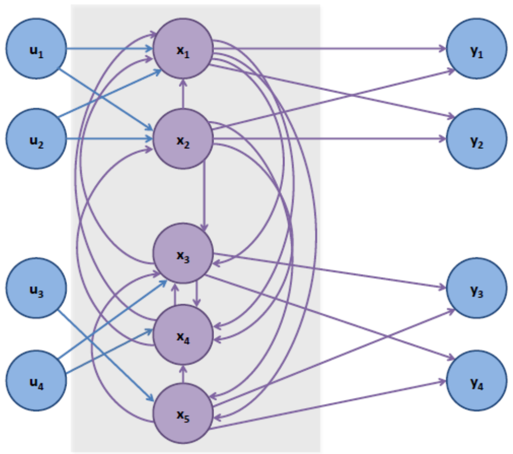
$$W(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$

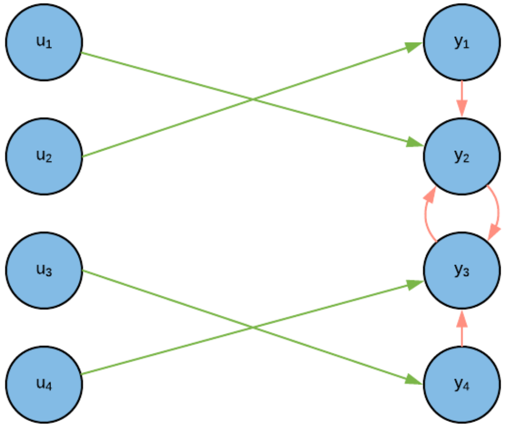
$$V(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}B_2 + B_1)$$

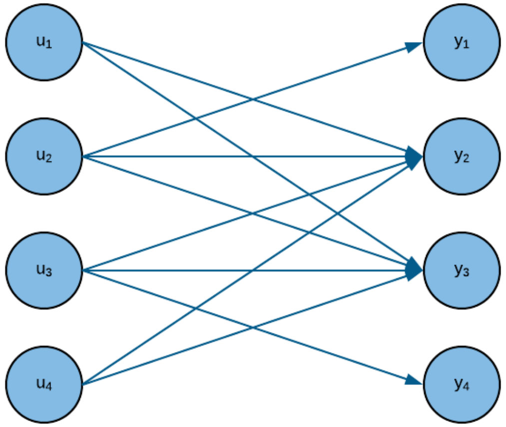
Problem Formulation

Zero State Equivalent and Structurally Equivalent Systems

System Representations







$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State Space Model

$$Y(s) = W(s)Y(s) + (s)U(s)$$

Dynamical Network Function

$$Y(s) = G(s)U(s)$$

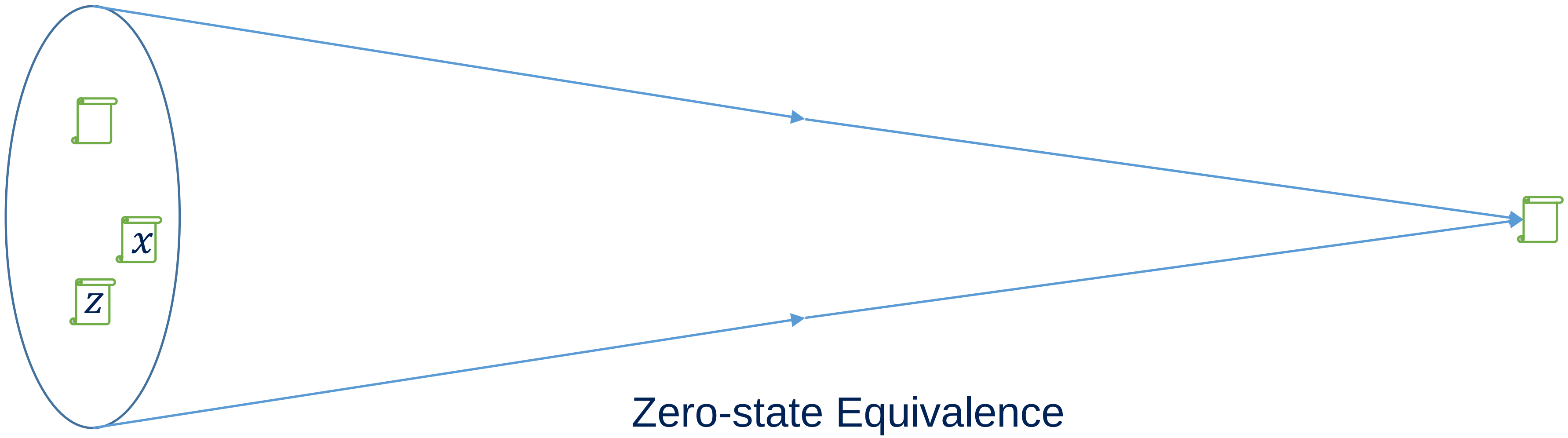
Transfer Function

$$W(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$
$$V(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}B_2 + B_1)$$

$$Y(s) = \left(I - W(s)\right)^{-1}V(s)U(s)$$

$$G(s) = C(sI - A)^{-1}B + \mathrm{D}$$

Zero State Equivalence



Zero-state Equivalence

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State Space Model

$$\exists T, z = Tx$$

$$\dot{x} = Ax + Bu \qquad \dot{z} = \bar{A}z + \bar{B}u$$
$$y = Cx + Du \qquad y = \bar{C}z + \bar{D}u$$

$$\bar{A} = TAT^{-1}, \bar{B} = TB, \bar{C} = CT^{-1}, \bar{D} = D$$

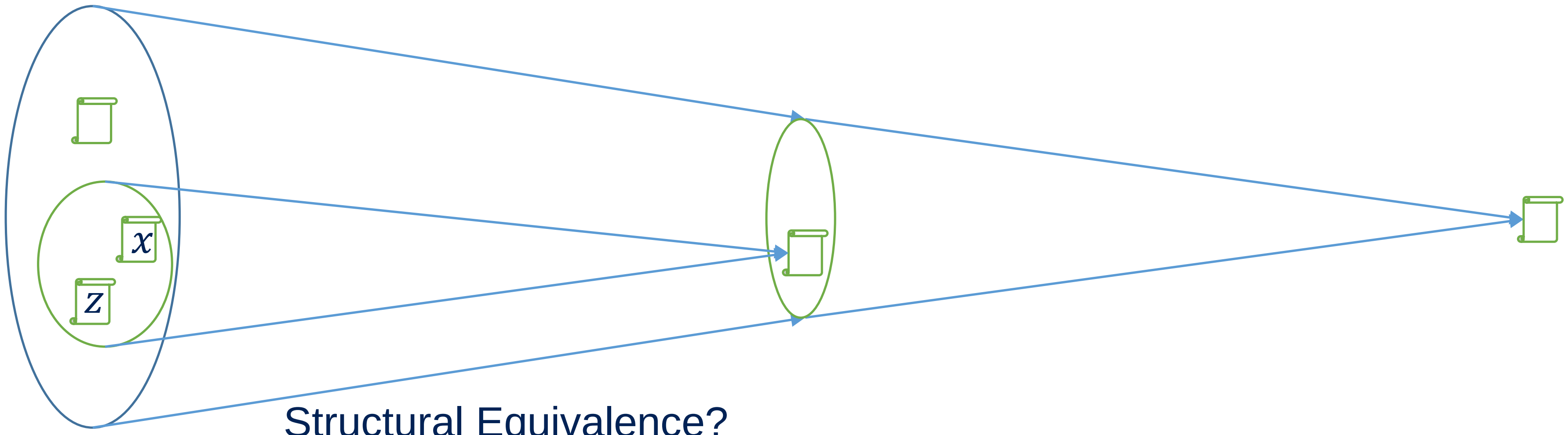
Algebraic Equivalence

$$Y(s) = G(s)U(s)$$

Transfer Function

$$G(s) = C(sI - A)^{-1}B + \mathrm{D}$$

Structural Equivalence



Structural Equivalence?

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State Space Model

$$Y(s) = W(s)Y(s) + V(s)U(s)$$

Dynamical Network Function

$$Y(s) = G(s)U(s)$$

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$$W(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$
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$$G(s) = C(sI - A)^{-1}B + \mathrm{D}$$

DNF Equivalence

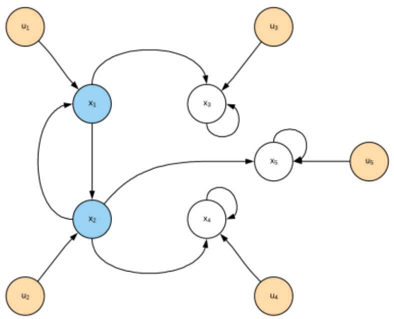
Definition 1 DNF Equivalence

Two State Space Realizations are DNF Equivalent if they are characterized by the same DNF (W, V) where:

$$W = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$

$$V = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}B_2 + B_1).$$

Example 1: Sufficient Condition of DNF Preservation

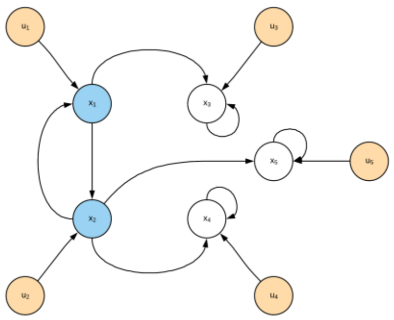


$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$

$$A_{12} = 0$$

Example 1: Sufficient Condition of DNF Preservation



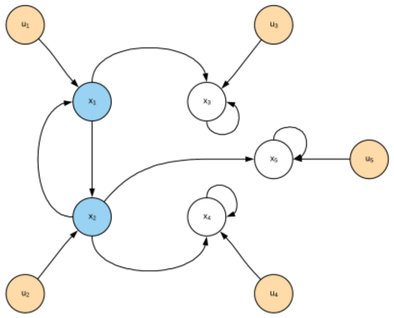
$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$

$$T = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$A_{12} = 0$$

Example 1: Sufficient Condition of DNF Preservation



$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

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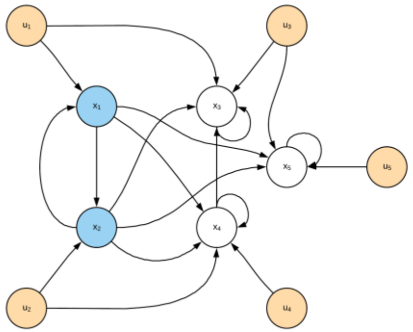

$$\bar{A} = \left[\begin{array}{cc|ccc} 0 & 1.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1.0 & 1.0 & 0 & 0 \\ -1.0 & 2.0 & 0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 0 & 1.0 \end{array}\right]$$

$$\bar{B} = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 1.0 & 0 & 1.0 & 0 & 0 \\ 0 & -1.0 & 0 & 1.0 & 0 \\ 0 & 0 & 1.0 & 0 & 1.0 \end{array}\right]$$

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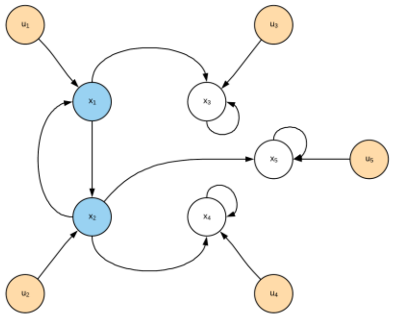
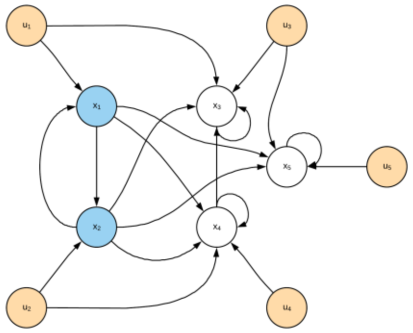
Example 1: Sufficient Condition of DNF Preservation



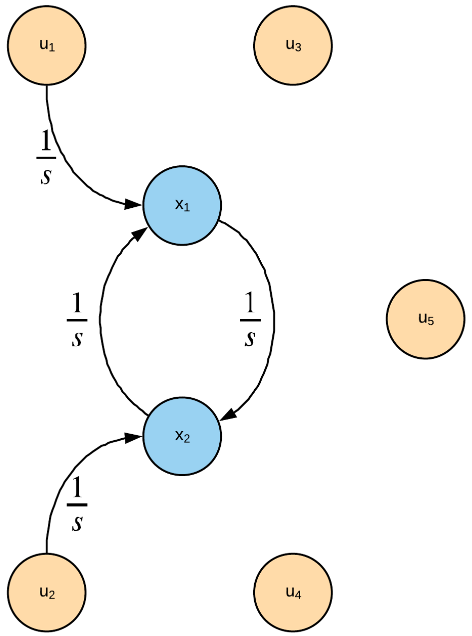
$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$



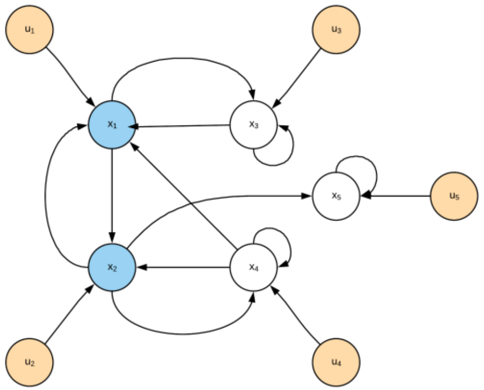
$$\bar{A} = \left[\begin{array}{cc|ccc} 0 & 1.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1.0 & 1.0 & 0 & 0 \\ -1.0 & 2.0 & 0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 0 & 1.0 \end{array}\right]$$

$$\bar{B} = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 1.0 & 0 & 1.0 & 0 & 0 \\ 0 & -1.0 & 0 & 1.0 & 0 \\ 0 & 0 & 1.0 & 0 & 1.0 \end{array}\right]$$



$$A_{12} = 0$$

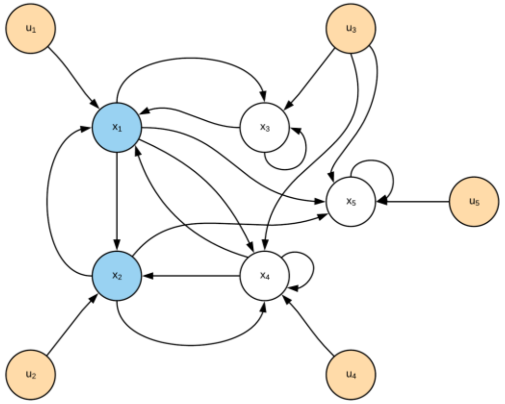
Example 2: Sufficient Condition of DNF Preservation



$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$



$$\bar{A} = \left[\begin{array}{cc|ccc} 0 & 1.0 & 2.0 & 1.0 & 0 \\ 1.0 & 0 & 0 & 1.0 & 1.0 \\ \hline 1.0 & 0 & 1.0 & 0 & 0 \\ -1.0 & 1.0 & 0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 0 & 1.0 \end{array}\right]$$

$$\bar{B} = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 1.0 & 0 \\ 0 & 0 & 1.0 & 0 & 1.0 \end{array}\right]$$

$$T_3 = 0$$

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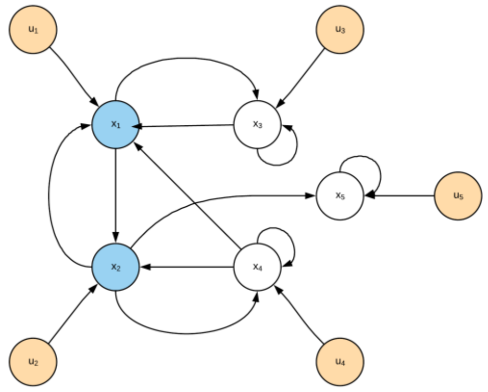
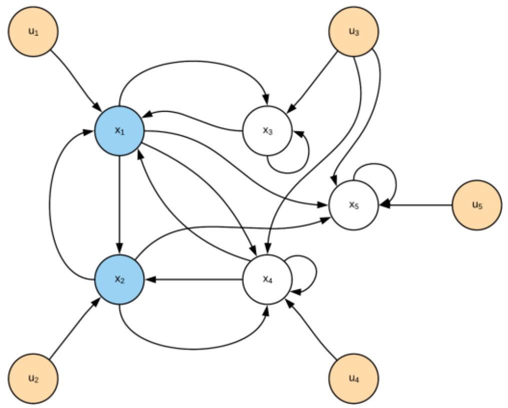
Example 2: Sufficient Condition of DNF Preservation



$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$



$$\bar{A} = \left[\begin{array}{cc|ccc} 0 & 1.0 & 2.0 & 1.0 & 0 \\ 1.0 & 0 & 0 & 1.0 & 1.0 \\ \hline 1.0 & 0 & 1.0 & 0 & 0 \\ -1.0 & 1.0 & 0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 0 & 1.0 \end{array}\right]$$

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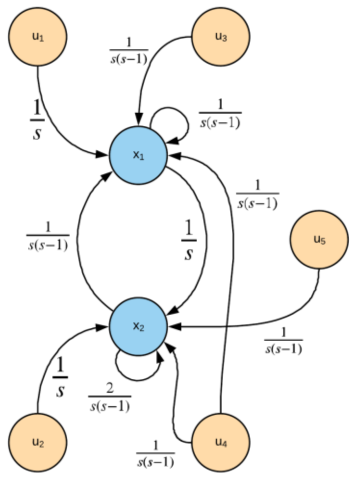

$T_3 = 0$

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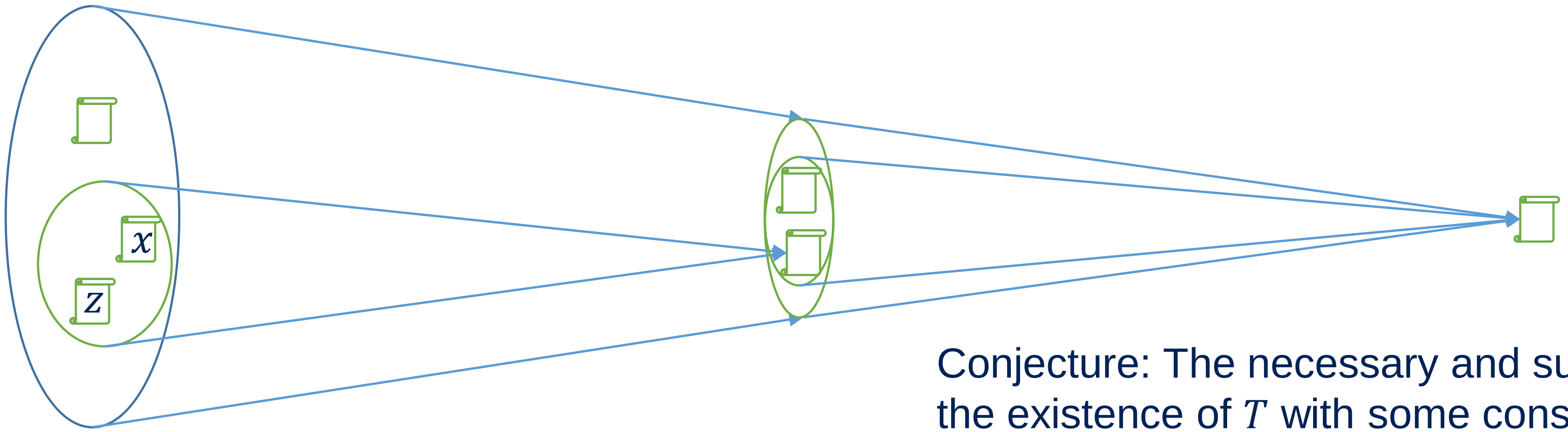
Pathway to Problem Formulation



Conjecture: The necessary and sufficient condition is the existence of T with some constraints.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$Y(s) = Q(s)Y(s) + P(s)U(s)$$

$$Y(s) = G(s)U(s)$$

State Space Model

Dynamical Structure Function

Transfer Function

$$W(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$
$$V(s) = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}B_2 + B_1)$$

$$Y(s) = \left(I - Q(s)\right)^{-1}P(s)U(s)$$

$$G(s) = C(sI - A)^{-1}B + \mathrm{D}$$

DNF Equivalence

Definition 1 DNF Equivalence

Two State Space Realizations are DNF Equivalent if and only if they are characterized by the same DNF (W, V) where:

$$W = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}A_{21} + A_{11})$$

$$V = \frac{1}{s}(A_{12}(sI - A_{22})^{-1}B_2 + B_1).$$

Problem Preserving DNF

What is the necessary and sufficient condition for preserving the DNF of a state space model $(A, B, [I \quad 0], 0)$ after a similarity transformation T?

Main Result

Necessary and Sufficient Condition For Preserving Dynamical Network Function

Main Result

Theorem 1

Necessary and Sufficient condition for preserving the DNF of a State Space model under similarity transformation.

A state space model $(A, B, [I\ 0], 0)$:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Main Result

Theorem 1 Necessary and Sufficient condition for preserving the DNF of a State Space model under similarity transformation.

A state space model $(A, B, [I\ 0], 0)$:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

And its transformed system $(TAT^{-1}, TB, [I\ 0], 0)$:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} I & 0 \\ T_3 & T_4 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ -T_4^{-1}T_3 & T_4^{-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Main Result

Theorem 1 Necessary and Sufficient condition for preserving the DNF of a State Space model under similarity transformation.

A state space model $(A, B, [I\ 0], 0)$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

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And a transformed system $(TAT^{-1}, TB, [I\ 0], 0)$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ T_3 & T_4 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ -T_4^{-1}T_3 & T_4^{-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

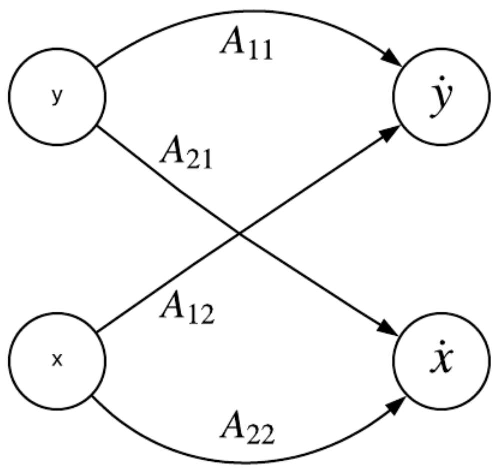
With $T = \begin{bmatrix} I & 0 \\ T_3 & T_4 \end{bmatrix}$, has the same dynamic network function (W, V) if and only if:

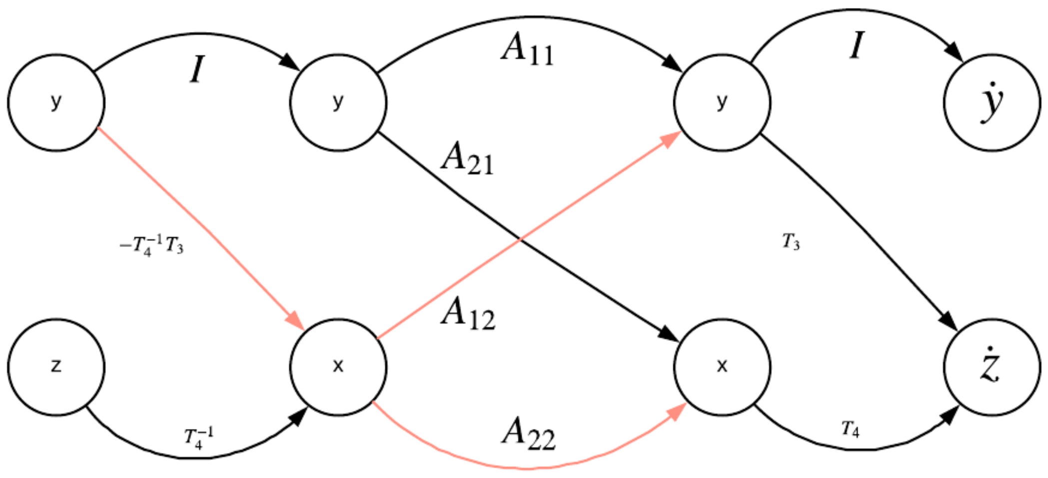
$$A_{12}(sI - A_{22})^{-1}T_4^{-1}T_3 = 0.$$

Explanation

$$A_{12}(sI - A_{22})^{-1}T_4^{-1}T_3 = 0$$

$$Y = WY + VU$$





$$\begin{bmatrix}\dot{y}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}y\\ x\end{bmatrix}$$

$$\begin{bmatrix}\dot{y}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}I & 0\\ T_3 & T_4\end{bmatrix}\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}I & 0\\ -T_4^{-1}T_3 & T_4^{-1}\end{bmatrix}\begin{bmatrix}y\\ x\end{bmatrix}$$

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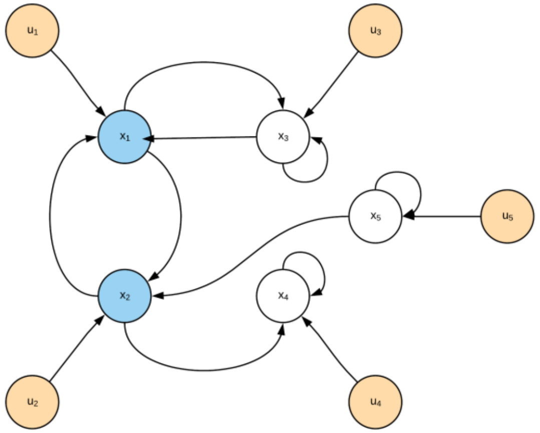
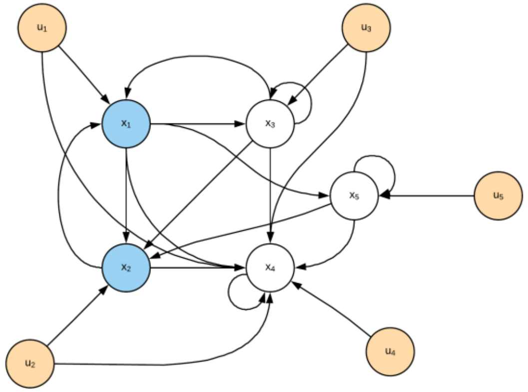
Example 3



$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$B = \left[\begin{array}{ccccc} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$

$$T = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$$



$$\bar{A} = \left[\begin{array}{cc|ccc} 0 & 1.0 & 1.0 & 0 & 0 \\ 1.0 & 0 & -1.0 & 0 & 1.0 \\ \hline 1.0 & 0 & 1.0 & 0 & 0 \\ -3.0 & 3.0 & 2.0 & 1.0 & -1.0 \\ 1.0 & 0 & 0 & 0 & 1.0 \end{array}\right]$$

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$$A_{12}(sI - A_{22})^{-1}T_4^{-1}T_3 = \frac{1}{s-1}A_{12}T_4^{-1}T_3$$

$$A_{12} \perp T_4^{-1}T_3$$

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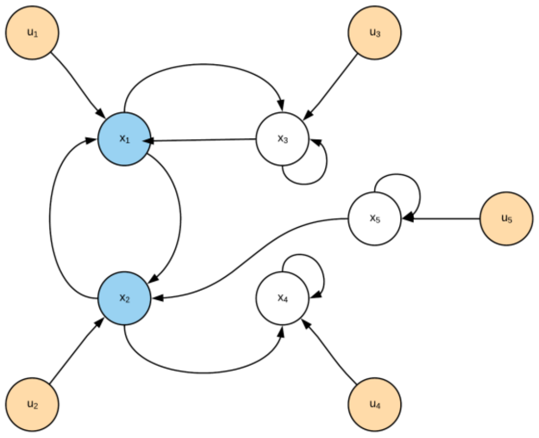

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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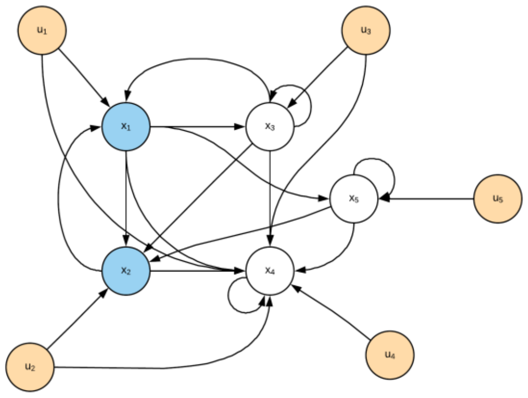

$$\bar{A} = \begin{bmatrix} 0 & 1.0 & 1.0 & 0 & 0 \\ 1.0 & 0 & -1.0 & 0 & 1.0 \\ 1.0 & 0 & 1.0 & 0 & 0 \\ -3.0 & 3.0 & 2.0 & 1.0 & -1.0 \\ 1.0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

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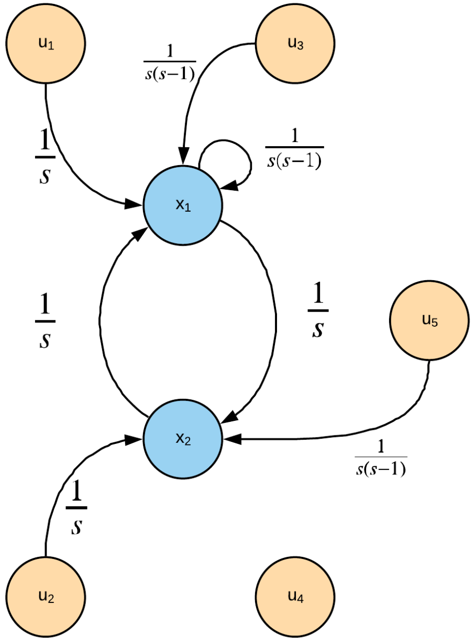

Implication

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Corollary 1 If A_{12} has full column rank, then all DNF equivalences are related by $T_3 = 0$.

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Proof: Since A_{12} has full column rank,

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Therefore, $A_{12}(sI - A_{22})^{-1}T_4^{-1}T_3 = 0$ only if $T_3 = 0$.

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Therefore, there exist a T_3 such that $A_{12}(sI - A_{22})^{-1}T_4^{-1}T_3 = 0$.

Future Work

- Develop a method for measuring likeness between state space models in terms of their dynamic network structure.
- Generalize the result to other type of dynamic network representations besides state space representation.
- Explore canonical forms from dynamic networks

Summary

Background

- Algebraic relationships between DNF, transfer function, and state space representation.

Problem Formulation

- DNF equivalent classes
- Examples: sufficient conditions for preserving DNF
- Problem Statement

Main Result

- Main Theorem
- Intuition
- Non-trial example where T3 not equal to zero

Implication

- Corollaries: conditions of existence of non-trivial DNF equivalent class.
- Future Works

Questions?