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On the Basics for Simulation of Feedback-Based Stock Trading Strategies: An Invited Tutorial Session

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Abstract— **This paper provides an overview of our CDC tutorial session covering the basics of simulation and performance evaluation associated with stock trading via feedback control methods. The specific trading algorithms which we describe fall under the umbrella of "pure technical analysis" in that they are model-free with no parametrization of the stock-price process assumed. True to technical analysis, we adopt the point of view that the stock price $p(t)$ is an external input with no predictive model for its evolution. The feedback controller adapts the investment level $I(t)$ based on the evolution of the trading gains or losses over time. In the introductory talk, it is explained how this point of view "opens doors" for new research contributions from the control community. The simulations which we consider are of two types: In some cases, the controller's performance is studied using synthetic classes of stock prices such as Geometric Brownian Motion. In other cases, real historical prices are brought into play. We refer to such a real-price simulation as a "backtest." Once we cover the trading mechanics, the notion of "benchmark price classes," data acquisition and coding of algorithms, the focal point becomes performance evaluation. That is, with $g(t)$ denoting cumulative gains or losses, we describe a number of metrics which are used to evaluate the performance associated with the trajectory pair $(I(\cdot), g(\cdot))$. Whereas the first half of the tutorial concentrates on trading a single stock, the last half addresses multi-asset portfolios, educational aspects and the notion of trading competitions.**

I. INTRODUCTION

The main objective of this tutorial session is to address a number of simulation or backtest issues related to stock trading strategies which are derived from control-theoretic considerations. We make the following distinction between a "simulation" and a "backtest." A *simulation* is carried out using artificial data such as Matlab-generated sample paths from a Geometric Brownian Motion. In contrast, a *backtest* is carried out using historical data such as daily closing prices and volume for some specific stock.

The tutorial begins with a very simple scenario: trading a single stock using a rather standard feedback control system evolving over time $t \geq 0$ with controlled input $I(t)$ being the investment, external uncontrolled input $p(t)$ being the price, account value $V(t)$ and cumulative gains or losses from trading the stock $g(t)$ over time interval $[0, t]$. In the later part of the session, we expand the presentation from performance evaluation for a single stock to portfolios. In this context, one highlight of the session is a presentation on "trading contests." This presentation describes how such contests should be organized and what supporting software is required. Our view is that such contests will not only drive future research but also serve as an excellent educational tool for graduate students conducting research in this area; e.g., see [45] and [46]. Via use of a virtual fund management system, it becomes possible to evaluate the performance of student traders in the context of control education.

1.1 Related Literature: We direct the uninitiated reader to a growing body of literature with various theoretical results involving financial markets with a control-theoretic flavor; see [1]-[34]. Some of these papers are only tangentially related to the material covered in this tutorial in that they deal with issues which are not directly related to the narrow focus here: trading a single stock or portfolio. Accordingly, for both pedagogical purposes and to create focus for this session, we subdivide the literature above into two categories: The first category, called *model-based approaches*, includes [1]-[25], and involves an underlying parameterized model structure which may or may not be completely specified. Depending on the degree of information about the parameters and disturbances, possibilities exist for robustness analysis, online identification, etc. The second category of papers, called *model-free approaches*, includes [26]-[34]. That is, the stock price $p(t)$ is viewed as an external input with no predictive model for its evolution. In particular, [26] and [27] analyze a "universal" portfolio strategy, while [28]-[34] focus on the adaptive properties of feedback control. Specifically, the latter references fall under the umbrella of "technical analysis" in its purest form. That is, no parameter estimation is involved and trade signals are generated based on some "pattern" of prices or trading gains.

While technical analysis is widely used in practice and being studied in academia, for example, see [35]-[42], when working in a model-free context, there is no formal theory which provides a sound theoretical foundation explaining why it works; e.g., see [36], [37] and [38] where this issue is discussed. Instead of a formal theory, in the literature, the case for the efficacy of technical analysis is made via empirical studies using historical prices and statistical analysis. The literature trail involving such empirical studies begins around 1992 with the results in [39] which are based on analysis of trading rules which trigger investment when the

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stock price $p(t)$ crosses some n-day moving average with n pre-specified in advance. On the heels of this work, we see a number of other significant empirical studies covering other types of trading rules and markets; e.g., see [40]-[42]. For a more detailed perspective on this entire line of research, the reader may wish to consult the introductory sections of the working paper [43]. In addition, additional details regarding the technical terms used in this paper can be found in [44].

1.2 From Theory to Practice: Other than providing the formulae necessary to carry out simulations and backtests, no theoretical justifications will be provided in this paper; i.e., proofs are included in the cited references. Our main objective here is to demonstrate how one takes a body of theory and builds a simulation or backtest which incorporates many practical considerations. For example, many papers in the literature pay little or no attention to important issues such as collateral constraints, margin calls, overnight price gaps and sudden changes in volatility. When conducting backtests and simulations, one of our main objectives is to determine if promising theoretical results "stand up" when subjected to these practical considerations. Our view is that strong performance of a feedback control strategy in a theorem-proving context is necessary but far from sufficient for prediction of success in real-world implementation.

1.3 Class of Control Strategies: To demonstrate how a theory-to-practice transition is carried out, we consider some technical analysis methods arising from classical feedback control loops. That is, no underlying stock-price model is assumed and no parameter identification is used. Instead, the trading algorithms dynamically modify the investment level $I(t)$ based on the profit-loss trend. Said another way, we adopt the point of view that $p(t)$ is to be treated as an external input with no predictive model for its evolution. Thus, in lieu of questionable predictive models, feedback induced robustness properties play a central role in determining investment strategies. The simulation methodology developed in this session is useful in a laboratory-type environment where performance and properties of strategies can be explored.

1.4 Benchmark Price Classes: Per discussion above, in this session, we focus on simulations and backtests driven by either empirical stock market data or from a class of artificially generated prices $p(t)$. We introduce the concept of a *benchmark price class* which is a set of price paths $\mathcal{P}$ endowed with various degrees of regularity. For example, a standard class of price processes used in finance is obtained from Geometric Brownian Motion (GBM); i.e., with constant drift μ and volatility σ, the underlying stochastic differential equation for the incremental returns on the price is

$$\frac{dp}{p} = \mu dt + \sigma dW_t$$

where W_t is a standard Wiener process which can be viewed incrementally in time as a normal distribution with zero as its mean and t as its variance; i.e., the probability density function at time t is

$$f_{W_t}(x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}.$$

An analogy can be made between benchmark price classes and benchmark problems in nonlinear optimization. When a new general-purpose nonlinear optimization algorithm is proposed, one usually attempts to "prove its worth" by testing it on various benchmarks. If an algorithm fails to perform on the benchmark class of convex functions, then one could argue that it should not be trusted for use on more general functions. On the other hand, if it can prove its performance over convex functions, one may then test its performance over additional benchmark problems, such as Rosenbrock's Banana function, the Six-Hump Camel Function, etc. We view trading strategies in a similar light. One must prove the performance of a strategy over benchmark price classes before it is to be trusted in real-world markets.

1.5 Discrete-Time Considerations: For the purposes of simulation, we consider a discrete-time trading environment. To illustrate using the price process above, once the period between trades Δt is established, the first step is to calibrate μ and σ on a per unit basis. Subsequently, the stock price $p(k)$ is updated from one time instant to the next where the discrete argument k is used to denote the time $k\Delta t$, measured in years; e.g., considering that there are about 252 trading days in a year, we take $\Delta t = 1/252$ when trading with daily closing prices. For example, with GBM prices we work with the equation

$$p(k+1) = \left(1 + \mu\Delta t + \sigma\sqrt{\Delta t}\epsilon(k)\right) p(k)$$

where $\epsilon(k)$ is a normal $\mathcal{N}(0,1)$ random variable. A benchmark price class is obtained by allowing (μ, σ) to range in some pre-specified rectangle in the plane. Subsequently, the goal is to provide results which are robust with respect to this class of allowed parameters.

A second example of a discrete-time benchmark price class is obtained by working with the *one-period returns*

$$\rho(k) \doteq \frac{\Delta p(k)}{p(k)} \doteq \frac{p(k+1) - p(k)}{p(k)}$$

and specifying admissible sequences $(\rho(0), \rho(1), ..., \rho(N-1))$ for simulation. For example, this class could be defined by the requirement $|\rho(k)| \leq \rho_{max}$ where $\rho_{max} > 0$ is given a priori as a limit on daily percentage changes. Again the key objective is to provide robust performance certifications with respect to this class. When trading is also included above, with investment $I(k)$ if we see a change in the stock price $\Delta p(k) > 0$ from k to $k+1$, the incremental contribution to the trading gains or losses is given by

$$\Delta g(k) \doteq g(k+1) - g(k) = \rho(k) I(k).$$

There are many other possibilities for benchmarks which one can entertain. For example, letting

$$R(N) \doteq \prod_{k=0}^{N-1} (1 + \rho(k)),$$

and noting that $R(N) > 1$ if and only if $p(N) > p(0)$, one can consider a *bullish* price class by enforcing the constraint $R(N) > 1$, a *bearish* price class with $R(N) < 1$ and a *round-trip* price class with $R(N) = 1$.

Finally, the reader is reminded that many benchmark price classes can be constructed using historical prices. For example, a benchmark test could involve a simulation involving adjusted daily closing prices for all S&P 500 stocks over some pre-specified period of time. By running simulations with various benchmark price classes, one may explore important aspects of trading strategies in controlled and pinpointed environments, allowing for a greater understanding and exploration of their characteristics.

1.6 Short Sale Mechanics: A real-world simulation program should also allow for the possibility of "short selling." By this, the following is meant: The trader has the ability to borrow shares from the broker and immediately sell them in the market in the hope that the price will decline. If such a decline occurs, at an opportune time, this short seller "covers" the position and realizes a profit by buying back the stock and returning the borrowed shares to the broker. Alternatively, if the stock price increases, sooner or later, the short seller buys back the borrowed stock at a loss and again "settles up" with the broker. In our feedback control formulation, this is all readily accommodated by allowing negative investment in the discrete-time trading equations. That is, if $I(k) < 0$, the equation for $\Delta g(k)$ above still holds; i.e., this quantity is positive on a price decrease $\Delta p(k) < 0$ and negative on a price increase $\Delta p(k) > 0$.

1.7 Other Session Considerations: An integral part of this tutorial will involve performance evaluation. That is, given a plot of system variables such as $I(k)$, $g(k)$ and $V(k)$, we describe the software implementation and monitoring of various performance metrics such as means, variances, Sharpe and Sortino ratios and drawdowns in account value; see Section 3 for details. Further to the evaluation of algorithmic results, the view which we express in this tutorial is that simulation should drive the development of the theory.

II. SIMULATION INGREDIENTS

Figure 1 shows a block diagram of the interconnections associated with feedback-based stock trading. At each time instant k, information such as price $p(k)$ and volume $v(k)$ is transferred from the broker to the trader. In turn, the trader uses this information to determine an investment level $I(k)$. This control signal $I(k)$ is then fed back to the broker who executes the requested transactions. We now outline the main "ingredients"which go into our simulations and backtests.

2.1 Price Data: Backtests involve the use of empirical stock market data as the external price input $p(k)$. There are various recognized sources for such data. For example, end-of-day closing prices, adjusted for splits and dividends can be downloaded for free from *Yahoo! Finance*. Additionally, a rather comprehensive database of historical prices at time scales from monthly to tick-by-tick is available from the *Wharton Research Data Services* for a subscription fee. In this tutorial, we focus on backtests involving daily closing prices that can be replicated using free data sources.

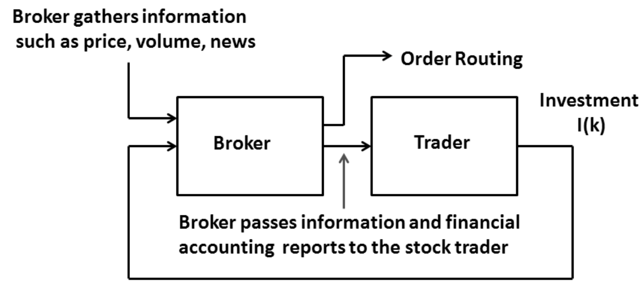

Figure 1: Feedback Loop with Broker and Trader

2.2 Information Transmission Between Broker and Trader: In addition to qualitative information such as stock ratings and news, most brokers also provide quantitative information such as earnings projections and balance sheets. As far as trade accounting is concerned, the broker provides frequent updates on the account value $V(k)$, gains and losses on stock positions $g(k)$ and stock prices $p(k)$.

2.3 Encoding of Feedback Control Law: A simulation or backtest program includes lines of code for the feedback control trading strategy which, for each portfolio asset, is a mapping from the outputs of the broker block to an investment amount $I(k)$. As a simple example of a feedback-based strategy, we consider a static gain control law in which the trader modulates the level of investment in a stock in proportion to the cumulative gains or losses from trading according to the formula,

$$I(k) = I_0 + Kg(k).$$

That is, the trader initially invests $I(0) = I_0$ in the stock and then begins to monitor the cumulative gain or loss $g(k)$ associated with this investment. Initially the gain is zero, i.e., $g(0) = 0$. However, over time, the position begins to either make or lose money depending on the movement of the stock. The broker passes this gain or loss information back to the trader, who subsequently adjusts the level of investment $I(k)$ in the stock according to the formula above, thus closing the loop from the trader to the broker. This is just one simple but important example of many possible trading schemes. To provide an additional example, one could consider a trader who wishes to limit the trade to some level $I_{max} > I_0$. In this case, the feedback loop would include a nonlinear saturation block and the update equation for investment would be

$$I(k) = \min\{I_0 + Kg(k), I_{max}\}.$$

The feedback loop configuration is seen in Figure 2 below.

2.4 Gain and Loss Accounting: As mentioned above, an important aspect of the broker block is that it keeps track of the gains, losses and overall account value of the trader. That is, at time k, the trader passes the dollar investment level $I(k)$ in a stock to the broker who executes any new order, and, upon receipt of a price update $p(k+1)$, computes the cumulative gain or loss $g(k+1)$ on that investment using the

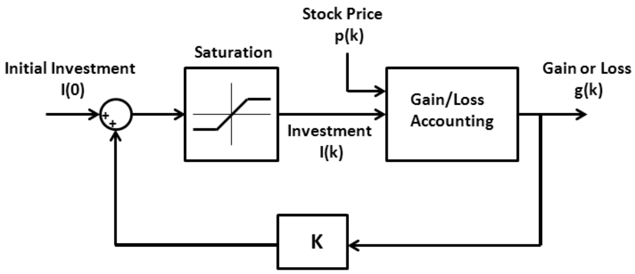

Figure 2: Feedback Loop with Saturation

increment $\Delta g(k) = (\Delta p(k)/p(k))I(k)$. That is, the change in the cumulative gain or loss $\Delta g(k)$ over a time increment is equal to the investment $I(k)$ multiplied by the return on the stock $\Delta p(k)/p(k)$ over the time increment.

When a portfolio of stocks is involved, the broker keeps track of the cumulative gain on each individual stock using the above equation. That is, assume the trader invests in n stocks where $p_i(k)$, $I_i(k)$, and $g_i(k)$ for $i = 1 \dots n$ denote the price, investment, and cumulative gain or loss, respectively. Then, the overall single-step trading gain is given by

$$\sum_{i=1}^{n} \Delta g_i(k) = \sum_{i=1}^{n} \frac{\Delta p_i(k)}{p_i(k)} I_i(k).$$

2.5 Interest and Margin Accounting: In addition to stocks, the trader's account may also contain idle cash or borrowed money from the broker on margin. That is, with

$$C(k) \doteq V(k) - \sum_{i=1}^{n} I_i(k),$$

the sign of this "account cash" surplus or shortfall quantity determines whether interest is accrued or margin charges need to be paid. Letting r_f and m denote the risk-free interest rate and broker margin rate for borrowed funds respectively for a period Δt, the broker also keeps track of interest earned or owed on cash; see below.

2.6 Account Value Reporting: Finally, the broker must report the entire value of the account $V(k)$ which is made up of the stock positions *plus* either idle or borrowed cash. Thus, to account for changes in the entire value of all stock positions and earnings/charges associated with cash, the broker performs the calculation,

$$\Delta V(k) = \sum_{i=1}^{n} \Delta g_i(k) + r_f \max\{C(k), 0\} + m \min\{C(k), 0\}.$$

These equations define the input to output dynamics of the broker block and are the basic equations involved in the simulation of a feedback trading strategy. In the $\Delta V(k)$ equation above, it should be noted that there is an implicit assumption that the trader can accrue interest on the proceeds of a short sale. In practice, this would only be true when the trader's account is sufficiently large. For a "small trader," the short-sale proceeds are generally "held aside" by the broker. Accordingly, in this case the cash balance $C(k)$ should be appropriately modified when updating the account value using the $\Delta V(k)$ equation.

2.7 Collateral Requirements and Margin Calls: When formulating the simulation model for trading, it is important to take account of the fact that the size of the trader's investment $I(k)$ is limited by the collateral requirements of the broker. For example, if this investment exceeds the account value $V(k)$ by too large an amount, transactions are "stopped" due to lack of collateral. In some cases, this problem is avoided if the account has a suitably large cash balance or if other securities in the account, not bought on margin, provide adequate collateral. In a brokerage account with total market value $V(k)$ at k, in simulations and backtests, we impose a constraint $|I(k)| \leq \gamma V(k)$ with $\gamma = 2$ being rather typical. Another possibility for inclusion in the simulations and backtesting is the issue of margin calls which can occur when the account value falls below the required collateral level. This includes the possibility of forced liquidation.

III. PERFORMANCE EVALUATION

The session will include an emphasis on measures of performance derived from the outputs of the simulations and backtests. When a strategy produces various time signals such as the gain/loss function $g(k)$, the account value $V(k)$ and the associated investment function $I(k)$, we describe measures which evaluate the "worthiness" of these results.

3.1 Classical Measures: First, the session will cover classical measures based upon first and second order moments of the return characteristics of the account value process. That is, we use the return over time period k,

$$r(k) \doteq \frac{V(k+1) - V(k)}{V(k)},$$

and the associated sample average and standard deviation of $r(k)$. A popular and classical performance metric based upon these quantities is the celebrated Sharpe ratio, for example see [47]. It is given by

$$SR \doteq \frac{\bar{r} - r_f}{s_r}$$

where r_f, as previously defined, is the risk-free rate of interest. The Sharpe ratio can be interpreted as a measure of risk-adjusted return, or expected return in excess of the risk-free rate per unit of standard deviation risk. Of importance is that the Sharpe ratio uses only first and second order moment information of the account value return distribution.

3.2 Measures Related to Use of Feedback: When feedback is used to modulate the investment $I(k)$, the distribution for gains and losses can become highly skewed; see [33]. Accordingly, in the session, we also pay attention to measures such as drawdown, skewness, and Sortino ratio, to name a few. That is, such measures capture information beyond the second order characteristics of the return distribution. As one example, consider the percentage drawdown at time k, defined as the current amount of percentage loss with respect to the previous peak of wealth. That is, the *percentage drawdown* of the account value at time k is computed as

$$d(k) \doteq \max_{j=0,1,\ldots k} \frac{V(j) - V(k)}{V(j)}.$$

Note that the drawdown is a function of time, and thus it is common to report its maximum

$$d_{max} \doteq \max_k d(k).$$

In the context of feedback-based trading strategies, it is argued that these alternative measures are more appropriate than classical metrics due to the highly non-Gaussian nature of the generated returns; see [33] and [34].

IV. TRADING CONTESTS AND EDUCATION

The presentation in this part of the tutorial is related to the previously discussed notion of "benchmark price classes." That is, performance is simply evaluated against the benchmark price classes, many of which are comprised of historical data. This idea opens the door for a framework with participation by many traders. Equivalently, if a single trader carries out tests with many a multitude of different trading strategies, we can deem the one with the strongest relative performance as being "best."

4.1 Experimentally-Driven Education: Motivated by the ideas above, this part of the tutorial covers some of the results in [45] and [46] on the development of a virtual fund management system to benchmark the performance of various controllers. From an educational point of view, this setting is ideal because students can immediately participate without expending large amounts of time learning economics and finance; i.e., they simply "code up" their strategies and "play" against their peers. We view this as "experimentally-driven education" because it is natural to expect that a student whose relative performance is weak will have incentive to learn the underlying theory.

4.2 Virtual Fund Management System: This part of the tutorial will include a description of a virtual fund management system aimed at implementation of the ideas above. Each participant is viewed as an agent interacting with a central system which manages all players' fictitious investments. Per discussion above, no model or belief about stock price dynamics is assumed. A unique feature of this virtual market is that it uses a dynamic rating mechanism for investor performance that rewards intelligence over luck. The proposed rating system avoids the use of any model of stock price evolution, and instead relies upon a measure that adjusts for market share instead of risk. It is argued that this dynamic mechanism is "fair" and rewards investor intelligence over good luck, all without requiring a model for stock price behavior.

V. SESSION OVERVIEW

As a guide to the session, this section provides highlights of the talks to be provided. Each of the six talks will be twenty minutes so as to be synchronized with the standard scheduling for the conference.

Talk 1 – On the Basics for Simulation of Feedback-Based Stock Trading Strategies: B. Ross Barmish will cover a number of basic issues associated with setting up and running simulations and backtests. Topics include data acquisition, simulation engines, benchmark price classes, simulation ingredients such as interest, margin, collateral constraints, account value reporting and data snooping.

Talk 2 – On Performance Metrics for Feedback-Based Stock Trading Simulations: Shirzad Malekpour will describe a number of classical performance metrics used in the backtesting of strategies. Given that feedback-based trading strategies tend to produce heavy-tailed distributions of gains and losses, in addition to classical mean-variance considerations and other classical measures such as Sharpe and Sortino ratios, metrics such as drawdown, skewness, kurtosis, probability of winning and value-at-risk will be covered.

Talk 3 – On Backtesting Pure Gain and Proportional-Integral Stock-Trading Controllers: James A. Primbs will cover proportional and Proportional-Integral (PI) trading methods and then consider simulations involving trend following, moving averages, and technical analysis methods. The presentation will also include the Simultaneous Long-Short (SLS) method and a number of backtests.

Talk 4 – Exploratory Simulation Before Theorem Proving: B. Ross Barmish will describe interesting discoveries and unexplained anomalies encountered in backtesting and simulation. The presentation will use various case studies, such day trading versus night trading, nonlinear trading strategies, saturation and restart ideas, etc., to indicate how exploratory simulation and backtesting motivate new research problems.

Talk 5 – On Practical Portfolio Balancing Considerations via Feedback Theory: James A. Primbs will provide an overview of classical Markowitz portfolio theory versus methods based on feedback. Of interest will be the performance when correlations between stocks are uncertain and time-varying. Case studies will be used to illustrate the differences and potential pitfalls of each method.

Talk 6 – On Trading Competitions: Their Role in Control Education and Research: Sean Warnick will describe the mechanics of setting up trading competitions and their use as a virtual laboratory for control methods in finance. A fund management system employing a dynamic rating mechanism to differentiate between luck and skill will be detailed.

VI. CONCLUSION

The purpose of this tutorial session is to introduce the basic tools and techniques required for the simulation, backtesting and performance evaluation of feedback-based stock trading strategies. Additionally, opportunities for new control research and education will be discussed with virtual trading contests being highlighted.

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